Intermodality in Multimodal Learning Analytics for Cognitive Theory Development: A Case from Embodied Design for Mathematics Learning

Sofia Tancredi, Rotem Abdu, Ramesh Balasubramaniam, and Dor Abrahamson

Abstract Multimodal Learning Analytics (MMLA) grant us insight into learners' physiological, cognitive, and behavioral activity as it unfolds. In this chapter, we query the relations among modalities, *intermodality,* in the context of a design-based research program studying the relations between learning to move in new ways and learning to think in new ways. In the first part, we reflect on how different methods have afforded purchase on the investigation, development, and elaboration of theoretical claims about the *multimodal* enactment of cognitive events, culminating in the use of Recurrence Quantification Analysis (RQA) to quantify the microgenesis of stable new patterns in hand movement and gaze. In the second part, we analyze an RQA case study spanning across hand and gaze modalities to examine the emergence of *intermodal coordination* at a critical moment in the mathematical task. We conclude with implications and open questions around intermodality in embodied learning.

Keywords Multimodal learning analytics · Perception · Embodied cognition · Mathematics · Intermodality

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1 Introduction

The embodied turn in the cognitive sciences brought forth reconceptualizations of what it means to think and learn. *"4E"* philosophers propose that the mind is embodied, enacted, embedded, and extended through the body and environment (Newen et al., 2018). 4E cognition suggests that digital technology is fundamentally intertwined with physical and bodily configurations and social meanings such that users create and communicate meaning through interaction (Dourish, 2001). More specifically, enactivists posit that cognitive structures emerge through repeated patterns in the perceptual guidance of motor activity (Varela et al., 1991). From this view, learners' sensorimotor activity plays a defining role in their cognition. Multimodal learning analytics here offer themselves as indispensable; multimodal data are positioned to grant unprecedented access into cognitive activity.

To take an enactivist stance seriously is to reimagine pedagogy to explicitly cultivate new ways of perceiving and moving as new ways of thinking (Abrahamson & Sánchez-García, 2016). Such an approach radicalizes the historical view in cognitive-developmental psychology research that mathematical reasoning has roots in sensorimotor activity (e.g., Piaget, 1968; Steffe & Kieren, 1994; von Glasersfeld, 1987). Design-based research program *embodied design* (Abrahamson, 2009, 2014, 2015) imagines an enactivist pedagogy where teaching and learning are understood as the cultivation of new perceptual structures, seeking to ground concepts in students' existing perceptuomotor capacities. Embodied designs create the conditions for learners to discover new ways of moving that ground mathematical concepts. Drawing on cultural–historical psychology (Vygotsky, 1926/1997), embodied design activities introduce disciplinary and symbolic artifacts as resources for enhancing students' pragmatic, epistemic, and discursive activity. Through this process, students come to perceive what they do in ways that ground semiotic expression. Specifically, introducing mathematical instruments into embodied-design activities, supported by either human (Abrahamson et al., 2012b; Flood et al., 2020; Shvarts & Abrahamson, 2019) or artificially-intelligent tutors, steers students toward mathematical discourse, by which they perceive and adjust their actions in accordance with the target concepts (Abrahamson et al., 2011), articulate conceptual relations between different movement strategies (Abrahamson et al., 2014; Flood et al., 2016), shift to paper-and-pencil inscription and calculation (Bongers, 2020), solve pictorial tasks (Abrahamson & Howison, 2010), and achieve new insight on familiar content (Shvarts et al., 2021).

The Mathematics Imagery Trainer for Proportion (MIT-P) is one instantiation of embodied design. This chapter will trace the MMLA history of the MIT-P project, illustrating how different analyses afforded purchase on multimodal learning phenomena, iteratively informing the theorization of the relationship between movement and mathematical thought. We will focus on a current frontier: modeling system dynamics with Recurrence Quantification Analysis, discussing two studies applying this method: Tancredi et al. (2021), which tracks the dynamics of hand coordination in MIT-P problem-solving, and Abdu et al. (under review), which tracks the dynamics of eye movements in MIT-P problem-solving. Setting forth from these analyses, we query the multi in multimodality, beyond the phenomena arising in each modality towards centering the interactions between them. We present an in-depth case study looking closely across data from these analyses to examine the microprocesses of intermodal perceptuomotor learning.

1.1 Overview of the MIT-P Project

The MIT-P is a tablet-based math instructional design for learning proportional reasoning (Abrahamson & Trninic, 2011). As an embodied design, the MIT-P is built to foster a new way of moving through goal-directed action under designed constraints. The design also takes up sociocultural theory by introducing mathematical artifacts as resources to elicit culturally endorsed actions and discourse. The MIT-P pedagogical rationale realizes design principles of embodied interaction (Dourish, 2001) by privileging pre-reflective situated skill as the cognitive grounding of professional competence. In the activity, users manipulate two parallel bars on a touchscreen with their fingers. The bars start off red, and learners are tasked with figuring out how to turn them green and keep them green while moving their fingers. The bars turn green when the ratio of the left bar to the right bar is 1:2, that is, when the left bar is at half the height of the right bar (Fig. 1). The MIT-P is designed to productively disrupt the common “additive” assumption, often displayed by children first learning ratio, that the difference between two quantities should remain constant as the

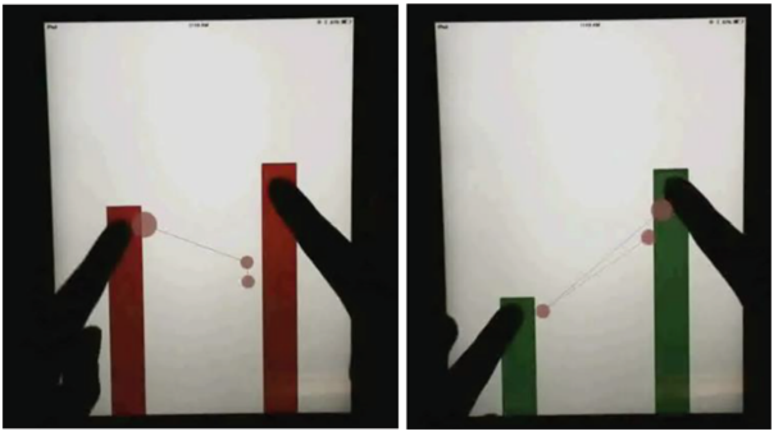

Fig. 1 Photos of user engaging with the MIT-P tablet application. In the first image, the bars are red because the encoded ratio, here 1:2, is not met. The bars turned green in the second example because the height of the left bar is in a 1:2 ratio with the height of the right bar. Note: the lines and dots between the bars represent an overlay added to the video in post-processing showing the participant’s eye tracking data

quantities increase, i.e., that a 1:2 ratio is equivalent to a 2:3 ratio and a 3:4 ratio. To fulfill the task directive of moving-in-green, users discover that instead of keeping the distance between their hands constant (instantiating an “additive” relation), they must instead increase this distance as the bars grow longer (instantiating a “multiplicative” relation).

Since its first release (Abrahamson & Howison, 2008), the MIT-P activity architecture has been evolving through a succession of design projects that used a range of technological platforms, from the mechanical to the mouse-driven to Wii, Kinect, and tablet, and working with PreK–16 participants in a range of settings (for implementation surveys, see Alberto et al., under review; Bakker et al., 2019). Some scholars working in classrooms have explored body-only non-technological activities (Kelton & Ma, 2018) and the learning of vicarious participants (Abrahamson et al., 2012a; Smith et al., 2016). Along the way, design changes were motivated by multiple considerations that emerged from empirical data analysis gathered in product evaluation studies, including insights into: the impact of media on user experience task design (Abrahamson & Howison, 2010); the relation of task choice to participation quality (Ba & Abrahamson, 2021); relations between interface imagery type and sensorimotor behavior (Rosen et al., 2016); the effects of discovery on learning (Abrahamson & Abdu, 2020); tacit rhythmic structure in the dynamics of students’ exploratory actions (Palatnik & Abrahamson, 2018); and teenagers’ cultural practices with interactive technology (Negrete et al., 2013). Products have expanded on the range of conceptual offerings (Shvarts et al., 2021) and figural variations on existing conceptual offerings (Duijzer et al., 2017).

1.2 Theoretical Framework: Intermodal Perception

Drawing on phenomenologist philosophers Husserl and Merleau Ponty, James Gibson (1966) defined perception as an active process that draws upon the relations among different perceptual organs to accomplish ecologically viable dynamic behavior. He wrote, “certain higher-order variables—stimulus energy, ratios, and proportions, for example—do not change” (p. 3). Eleanor Gibson (1969) elaborated upon this view by distinguishing several types of intermodality.[1] One is *intermodal transfer*, whereby perceptual discrimination abilities in one modality carry over in another modality. She gave the example of how a man who lost his sight at 10 months and recovered it over the age of 50 could recognize letters visually, which he had only previously experienced through the tactile modality. Notably, intermodal transfer can be inhibited by the presence of distinctive modality-specific

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[1] We use the term intermodality to signify the dynamic process of coordinating perceptual information across modalities. This usage is distinct from intermodality as multisensory integration (e.g., Ernst, 2008).

properties such as color (E. Gibson, 1969). Another type of intermodality is that of *amodal perceptual properties*. In contrast to modality-specific properties, such as the color blue, amodal properties are higher-order relational structures such as jerkiness, or a transition from rough to smooth. These properties are not modality-specific, although they reside in properties of light, sound, movement, or vibration. Perceptual development in Eleanor Gibson's model is a process of discovery of intramodal and intermodal invariant stimulus relations and transformations that become more specific and attuned to higher–order relations (E. Gibson, 1969).

The MIT-P is a fertile context for examining intermodality in that the activity of multiple systems participates and intertwines. Learners move their hands on a touchscreen, monitor their action visually, and dialogue with a tutor. Bodily participation of the hands and eyes is sensory and motor: the hands perform motor action *and* provide kinesthetic-proprioceptive feedback to the learner. Similarly, eye gaze detects color-feedback, supports visual proprioception (J. Gibson, 1966) of the hands' positions, *and* moves in strategic, goal-directed ways to participate in the control of action. We begin by tracing strands of research investigating each of these modalities in the MIT-P project, showing how these threads converge towards an intermodal research agenda.

2 Multimodal MIT-P Analyses: A Brief History

As a design-based research project, the MIT-P has gone through numerous iterations and forms eliciting evolving analyses that feed back into theorizing learning. Data collected from users of the MIT-P include eye tracking, tablet data, transcripts, and video. Early work on the MIT-P project used detailed qualitative analysis of video and eye-tracking data to study the microgenesis of focal movement forms. Such analyses have been pivotal in identifying how learning unfolds in the MIT-P, including the strategies learners use (Abrahamson et al., 2014) and the role and activity of the tutor (Abrahamson et al., 2011; Flood et al., 2020; Shvarts & Abrahamson, 2019). Later work took up a range of quantitative methods including statistical methods and machine learning to detect participants' strategies. Below, we describe the methods and findings of studies pertaining to the kinesthetic and visual modalities' role in MIT-P learning.

2.1 Hand Movements

Touchscreen data have been effectively used to identify learner strategies. Machine learning was used to differentiate between sequential "A per B" strategies that focused on the relative displacement of each hand and "speed" strategies that focused on how one hand moved faster than the other (Pardos et al., 2018). These hand-movement strategies offer different entry-points into mathematical discourse about the activity. Other researchers applied machine learning to develop an intelligent tutoring agent that responded dynamically to MIT-P learners' needs (Abdullah et al., 2017).

Statistical analyses of MIT-P hand data have highlighted regime-switching dynamics within and between participants (Ou et al., 2020a, b). These analyses used a mixture Regime-Switching Hidden Logistic Transition Process to identify characteristic hand movement-language transcript regimes and assign students to clusters according to their regime transitions. Participants were found to cluster as quick or slow discoverers. Within subjects, participants moved through three stages: an initial stage characterized by hands moving at the same heights, an intermediate stage of exploring different hand relationships, and a final stage of moving-in-green where the hands maintained the target ratio, with occasional relapses into Regime 2. These findings suggest that learners transition between stages with different characteristics in this embodied design environment, reminiscent of phase transitions between different stable regimes (Ou et al., 2020a, b).

2.2 Eye Movements

In parallel, eye tracking studies of MIT-P data have revealed critical insights about student strategy not available to tutors in real-time. Qualitative eye-tracking studies found that over the course of the task, participants began to shift their gaze towards new areas of interest beyond their fingers, such as the projection of the left bar on the middle of the right bar (Abrahamson et al., 2016; Shayan et al., 2015). These gaze patterns, together with analysis of participant's verbalizations about their actions, suggested that participants were developing *attentional anchors* supporting the transition from additive to multiplicative movement and reasoning. Attentional anchors (Hutto & Sánchez-García, 2015) are perceptual objects that come forth to facilitate motor action. The anchors are real or imagined relational features that can be manipulated to control activity. For example, dancers might imagine being pulled by a string emerging from the top of their head to control multiple aspects of their posture. Work in cognitive psychology (Mechsner, 2003, 2004) corroborates that the enactment of complex motor actions is often enabled by the perception of Gestalt organizing structures. In the context of the MIT-P, learners were found to exhibit idiosyncratic attentional anchors supporting moving-in-green such as the interval between their fingers, or imaginary lines connecting features on the screen (Abrahamson et al., 2016). Duijzer et al. (2017) corroborated and further described the emergence of attentional anchors quantitatively, analyzing the frequency, duration, visit count, and fixation count of area-of-interest gaze patterns alongside lag-sequential analysis of language transcripts. Eye-tracking study findings mark the importance of multimodality in the MIT-P context: hand positions alone do not capture the critical changes that allow learners to achieve fluency with the task.

2.3 RQA Analysis

Analyses of hand movements suggested the presence of phase transitions between different regimes as learners engaged with the MIT-P task. To further investigate this phenomenon, a next set of analyses turned to dynamical systems theory (Kostrubiec et al., 2012; Thelen & Smith, 1994) and the related nonlinear analysis method of Recurrence Quantification Analysis (RQA) to describe the dynamics of the different stages present in MIT-P problem-solving. We pause here to briefly introduce RQA as context for our RQA-based case study in the coming section before describing MIT-P work with this method to date. RQA is a nonlinear method for studying the structure of a dynamical system by detecting repetition patterns (Marwan et al., 2007). RQA is particularly apt for the study of dynamical systems in that (1) RQA does not assume linearity, allowing for the study of complex systems with interaction-dominant dynamics, and (2) RQA treats variability as a quality of the system rather than as noise, making it powerful for understanding noisy, nonstationary data (Webber & Zbilut, 1994). RQA begins with the construction of a recurrence plot, a plot that compares every state in a time series to every other. These plots can be constructed for an individual time series, quantifying its dynamics, or a pair of time series, quantifying the degree and stability of these systems' coupling. Features of the resulting recurrence plots are then quantified, such as the proportion of matching states (recurrence rate), percent of points falling on diagonal lines indicating a repeated sequence (determinism), the average length of a repeated sequence (meanline), distribution of repeated sequence lengths (entropy), and duration of continuous repetitions (trapping time). These metrics reflect the repetition, predictability, stability, disorder, and duration of connected states in the time series. There are several types of RQA, including auto-RQA (*aRQA*), which examines self-similarity within a single time series, cross-RQA (*cRQA*), which examines alignment between two time series (e.g. time series 1 and time series 2 were in the same state), and multidimensional RQA (*MdRQA*), which looks at the recurrence of states across multiple systems (e.g. patterns of repetition in sets of states defined by the states of multiple time series) (Amon et al., 2019).

RQA's is well-suited to enrich MMLA, especially in the analysis of time-bound data streams. Applications thus far include work by Allen et al. (2017) showing the promise of RQA's attention to temporality for natural language processing, wherein they leveraged RQA of word use to develop a more robust model for predicting reading comprehension. Additionally, RQA has been shown to work well with most time series data, which places no restrictions on the statistical distribution of data or on data set length. RQA provides a characterization of a variety of features of any given time series, including a quantification of deterministic structure and of nonstationarity, particularly useful in the context of MMLA.

Originating in physics and consequently applied in the study of physiology, cognition, joint action, economics, and communication, RQA remains rare in math and science education research, with a handful of notable exceptions (e.g., Fleuchaus et al., 2020; Stephen et al., 2009). The MIT-P has been the first context for the application of RQA to embodied design data. A first study (Tancredi et al., 2021) modeled the dynamics of the bimanual system using RQA as learners discovered and began to move fluently in green. A second (Abdu et al., under review) modeled the dynamics of the gaze patterns in relation to stages of hand movement. Both RQA studies segment participants' time series according to stages appearing in most interviews: initial Exploration searching for green, Discovery, spending substantive time in green positions, and finally, Fluency, moving both hands simultaneously while keeping the bars green.

Hand cRQA[2] The first relationship of interest in the MIT-P context was that between the hands. The ratio of the left hand to the right is the conceptual core of the MIT-P activity. How do the nonlinear dynamics of this core bimanual coordination pattern evolve? Whereas previous analyses of hand data identified strategies or stages, RQA enabled comparison of hand coordination within each of these stages in terms of their stability, predictability, order, and duration of connected states. In a cRQA analysis of the left- and right-hand data (Tancredi et al., 2021), we found that when participants reached Discovery, they exhibited an increase in the RQA determinism metric, reflecting an increase in the predictability and coupling of the hands. When participants reached Fluency, this yielded a leap in RQA metrics including recurrence rate and meanline, reflecting an increase in coupling, stability, and predictability. These findings corroborate that a qualitative change arises between finding-piecemeal-greens and moving-in-green through which a new, more stable, more predictable coordination emerges.

Gaze aRQA RQA analysis of bimanual data showed abrupt reconfiguration into "moving-in-green." Do we see similar changes in gaze patterns? How might gaze be bound up in the gradual constitution of bimanual coordination? Is pattern emergence an independent, unimodal phenomenon? Another study (Abdu et al., under review), examined the sequence of gaze fixations in areas of interest corresponding with the dynamic MIT-P solution space over the course of the activity with categorical RQA. Whereas prior gaze analysis unveiled types of patterns in the data overall, RQA was able to shed light on the dynamics of these patterns within each stage of hand coordination. This permitted looking across participants' idiosyncratic solutions to identify a common quality of fluent performance: gaze-pattern dynamic stabilization. Abdu et al. (under review) found a decrease in entropy during the Discovery stage indicating an increase in the level of order in gaze behavior after finding green, as well as an increase in recurrence rate, determinism, meanline, and trapping time in the Fluency stage. These findings showed that gaze patterns were

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[2] cRQA quantifies dynamical aspects of the coordination of two time series (here, the left- and right-hand position time series). aRQA quantifies recurrent patterns within a single time series. The gaze data are categorically coded according to areas of interest relative to the hands such as the top of the left bar or the middle of the right bar (see Abdu et al., under review). Thus, cRQA analysis compares the continuous hand position time series, whereas the categorical aRQA compares the categorical gaze time series to itself.

more ordered in Discovery than in Exploration, and more stable, repetitive, and predictable during Fluency than in prior stages.

Hand and gaze RQA analyses validated the distinct dynamics of Exploration, Discovery, and Fluency stages of the MIT-P task for both hand coordination and gaze patterns. The convergence of both hand and gaze dynamics towards greater stability in Fluency suggests that the dynamics of these two systems may be related. We will investigate this hypothesis in the next section by juxtaposing the moment-to-moment progression of hand and gaze dynamics for a focal participant.

3 From Multimodal Gaze and Hand Movement to the Intermodal Emergence and Stabilization of Attentional Anchors: An RQA Case Study

The diverse multimodal analyses of MIT-P data point to the interactions between hand and gaze as critical to the process by which learners increase their grip on the problem space, culminating in mathematical insights. We will delve into the case of a single participant appearing in both the Tancredi et al. (2021) and Abdu et al. (under review) analyses.

3.1 Research Question

As design-based researchers of mathematical cognition, teaching, and learning, we are hoping to gain deeper understanding of relations between features of interactive technologies and the micro-learning processes they enable. Embodied designs engage learners through multiple modalities, each with their own dynamics. This case study seeks to investigate how the dynamics in each modality relate to one another and to the broader emergence of fluent behavior: how do the relations between the visual and kinesthetic modalities change as a prototypical participant gains fluency with moving multiplicatively?

3.2 Methods

Research Design We conducted a mixed-methods analysis of multimodal data from one participant's semi-structured clinical interview learning with the MIT-P.

Participants The case study participant was one of 39 grade 5 and 6 participants in the Netherlands trying the MIT-P application for the first time (Duijzer et al., 2017). We selected this participant, pseudonymed Finn, because their overall trends in RQA mirrored cross-participant trends for both hand cRQA and gaze aRQA.

Procedure This is a secondary analysis of data collected through semi-structured clinical interviews (Duijzer et al., 2017). A researcher-tutor led the interviews. Her role was to reiterate task directives, encourage exploration, and prompt the participant to think aloud and explain their actions on an ongoing basis. We focus on the first stage of the interview, in which participants first learned the multiplicative movement pattern.

Measurement Multiple data streams were recorded from these interviews: eye–tracking data using Tobii x2–30, video recordings taken from the participant's perspective, and audio recording of participants' dialogue with the interviewer.

Data Preparation and Analysis To contextualize results within the student's specific learning trajectory, we began with a qualitative analysis of the audio-video recording. We also calculated the correlation between each hand's y-location and y-axis gaze position in each task stage as an initial reflection of the relationship between these two modalities. For a more in-depth analysis of this relationship, we generated windowed RQA plots for this participant's categorical gaze time series and for their two-finger touchscreen locations over time. We focused on the RQA metrics of recurrence rate, determinism, and meanline for this analysis. The windowed plots (Figs. 5, 6 and 7) consist of the three RQA metrics taken for a window of the coming 50 seconds, sampled for every second of the time series (lag width = 5). Gaze data were coded according to 14 areas of interest: the top, middle, and bottom of the left and right bar, the space between the bars segmented according to the same divisions as the right bar, and the spaces above each bar (coding detailed in Abdu et al., under review). We added a graph of hand heights over time to figures and overlaid the transitions from Exploration to Discovery (0:39) and Discovery to Fluency (3:42) identified in Abdu et al. (under review) onto the windowed plots for reference in connecting the case study to findings from these prior studies.

3.3 Results

Overview: The Case of Finn We describe Finn's learning process to contextualize our analysis within the broader task-interviewer-environment system.

When the task began, Finn found green almost immediately (Fig. 2a) while gazing at his right finger (0:00–0:09). He then raised both hands with the right slightly higher than the left, keeping a roughly constant distance between the bars and maintaining his gaze on the right bar (Fig. 2b) (0:09–0:31). He reset his fingers to the bottom of the screen and began to raise them together, looking at the tops of the bars (Fig. 2c) (0:31–0:41).

Finn transitioned into the Discovery stage by holding his hands in an identified green position, gazing at the top of the left bar and just below the top of the right, drawing his eyes from one to the other almost horizontally (Fig. 3a) (0:41–1:19). He

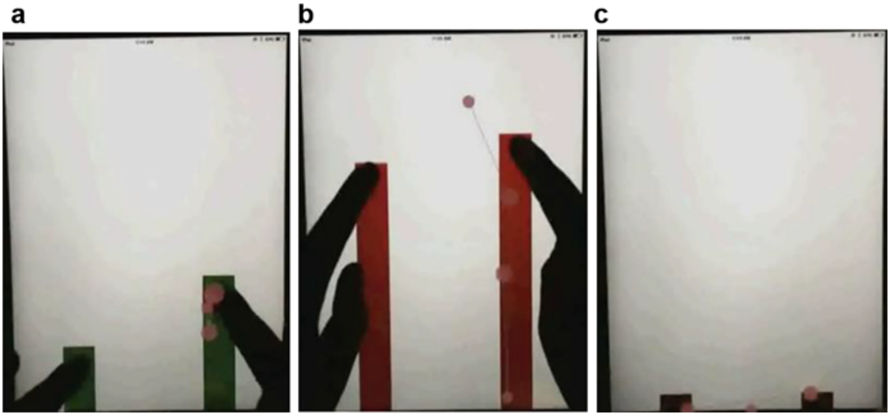

Fig. 2 Finn's activity during the Exploration stage. (**a**) Finding a first green location. (**b**) Raising fingers with a set distance between them. (**c**) Resetting to bottom of the screen

moved his fingers slightly up and down, exiting and re-entering a green position and repeating the visual pattern. At this stage, Finn articulated that to make green, "you have to put both of them on a certain thing that turns green." Then, to illustrate this point (1:19–1:32), Finn inverted which bar was taller (Fig. 3b), commenting: "If you put one here and one goes down, then it doesn't turn green," then restoring the right-bar higher position, "but if I keep this one here, it will be" (Fig. 3c). Prompted to find other greens, Finn tried raising his hands with a set distance between them, eventually adjusting his left hand downwards to find another green (Fig. 3d) (1:32–1:41). During this time, Finn displayed a range of different gaze patterns, prominently featuring finger to finger and left finger to below the right finger. Finn then alternately raised the left and right bars higher (Fig. 3e and f), frequently gazing not only at the fingers but also at the halfway projection of the shorter bar on the taller one. He found a series of green positions this way (1:41–3:42). The tutor then prompted Finn to try moving such that the bars stayed green continuously (3:42).

After this prompt, Finn initiated his most fluent performance of movement-in-green (3:42–5:07), marking the Fluency stage. Finn found an initial green and looked from finger to finger (Fig. 4a). He then began to move slowly up the screen, moving his right and left fingers in sequence. As he did so, in addition to the tops of each bar, Finn's eyes frequently jumped to the middle of the taller right bar (Fig. 4b and c). When asked to explain what he was doing, Finn responded, "I think there must be a certain distance."

Hand-Gaze Correlation From Exploration to Discovery to Fluency, hand heights and gaze heights became increasingly correlated. Initially, during the Exploration stage, there was no correlation between hand heights and gaze height (left hand: $r\ (225) = -0.002$, $p = 0.979$; right hand: $r\ (233) = -0.017$, $p = 0.792$). During the Discovery stage, there was a moderate correlation between hand heights and gaze height (left hand: $r\ (1427) = 0.360$, $p < 0.001$, right hand: $r\ (1436) = -0.017$,

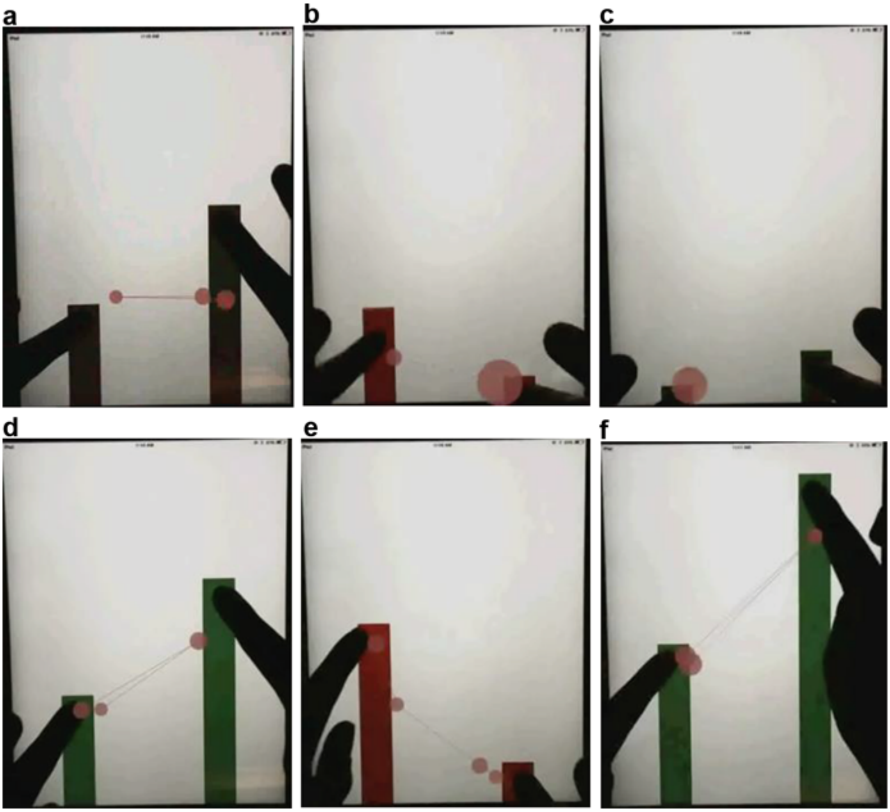

Fig. 3 Finn's activity during the Discovery stage. (**a**) Holding green and drawing gaze from top of left bar to midway up right bar. (**b**-**c**) Finn shows the interviewer how a left-higher position does not turn green whereas a right-higher position does. (**d**) Finn works to find other greens, moving his hands at a set distance and adjusting. (**e**-**f**) Finn continues to try left-higher and right-higher positions, often gazing at the fingers and a point below the higher finger

$p < 0.001$). During the Fluency stage, there was a high correlation between hand heights and gaze height (left hand: r (802) = 0.697, $p < 0.001$; right hand: r (803) = 0.069, $p < 0.001$). Variance in hand and eye movements became more related over the course of the task.

3.3.1 RQA Analysis

Recurrence Rate Gaze auto-recurrence rate indicates the frequency with which gaze states (here, specific areas of interest such as the top of a bar) are repeated. Hand cross-recurrence rate indicates the degree of coupling between the right- and left-hand heights. Figure 5 shows sliding window plots of change over time in the gaze (a) and hand (b) recurrence rates as the hands explored different positions

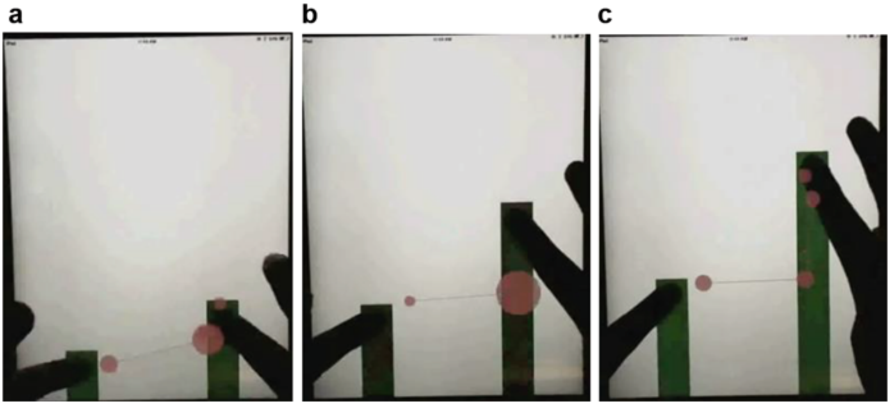

Fig. 4 Finn's activity during the Fluency stage. (**a**) Finding an initial green position. (**b**-**c**) Gradually moving up the screen, adjusting to keep the bars green. Gaze patterns include the midpoint of the right bar

(c). Finn's trajectory showed a high-low-high pattern of hand coupling (Fig. 5b). An initially stable coordination pattern (Fig. 5b-1) destabilized for a period of low coupling between the hands (Fig. 5b-2) before culminating in a new stable coordination pattern (Fig. 5b-3 and 4). In contrast to the hands, the gaze started out with low recurrence and maintained this throughout the Discovery stage (Fig. 5a-1 and 5a-2). At the end of the Discovery stage (Sect. 3), both gaze recurrence rate and hand coupling[3] increased sharply, yielding high recurrence in both modalities in the Fluency stage (Fig. 5a-4 and 5b-4).

Determinism Determinism captures the predictability of a system. Figure 6 shows sliding window plots of change over time in the gaze (a) and hand (b) percent determinism as the hands explored different positions (c). Finn's hands began highly deterministic (Fig. 6b-1 and 6b-2), became less deterministic through the Discovery stage (Fig. 6b-2, 6b-3, and 6b-4), and then once again reached high levels of determinism (Fig. 6b-5). During most of these first two stages, Finn's learning trajectory showed distinct determinism patterns in hand and gaze. During stages when hand determinism was high (Fig. 6a-2 and 6b-2), gaze determinism was low, and vice versa (Fig. 6a-3 and 6b-3). These divergent dynamics resolved in Sect. 4 as both the hands and gaze exhibited a decrease in determinism (Fig. 6-4), followed by high levels of determinism through to the end of the task (Fig. 6-5).

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[3] The brief drop in hand coupling at the end of the Discovery stage is likely a by-product of the participant attempting to generalize the ratio to an inverted position where the left hand was above the right.

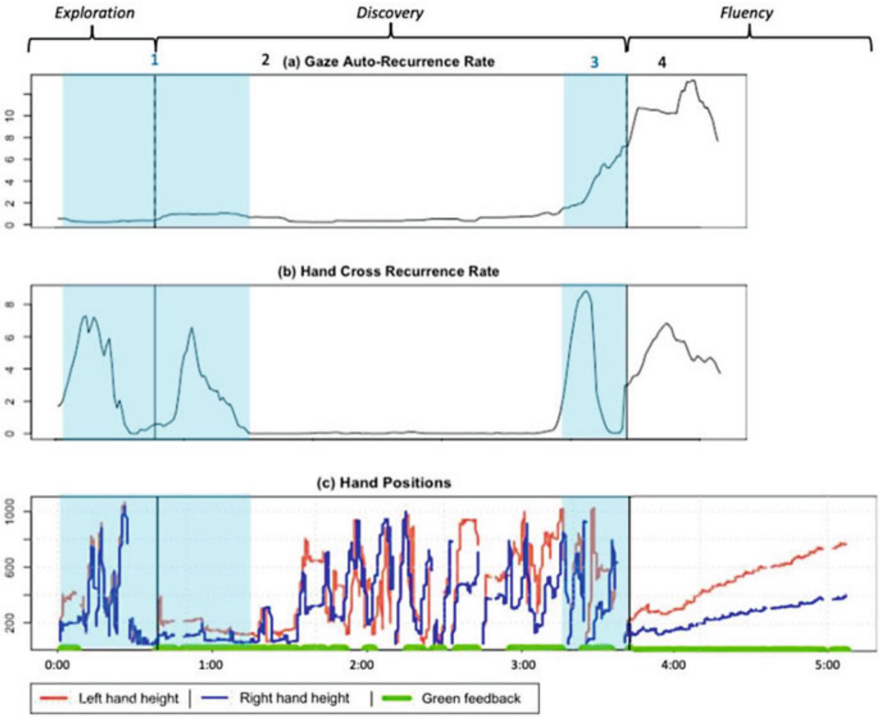

Fig. 5 Sliding window plots of hand and gaze recurrence rates over time (window size = 50 seconds, step size = 1 second). Time series of hand positions included for reference. (**a**) Gaze auto-recurrence rate over time. Recurrence rate starts low (<2%) and increases at the end of the Discovery stage to greater than 10%. (**b**) Hand cross-recurrence over time. Recurrence rate starts high, drops to close to 0% during Discovery, and increases at the end of the Discovery stage. (**c**) Plot of right-hand (blue line) and left-hand (red line) heights over time during the task. Green indicates moments when the participant fulfilled the target ratio of heights between their hands. All three graphs share the same time × axis. Alternating blue and white shaded areas 1–4 highlights five qualitatively distinct portions of the time series for ease of reference in the text

Meanline The meanline RQA metric captures the stability of a system. For the gaze aRQA, this metric reflects the mean duration of repeated sequences. For the hand cRQA, this metric reflects the mean duration of coordinated bimanual actions. Figure 7 shows sliding window plots of gaze (a) and hand (b) meanline relative to hand positions over time (c). Hands and gaze started off with high stability (Fig. 7-1). As the participant discovered greens and lingered in them (Fig. 7-2), the gaze destabilized. As the participant attempted to move-in-green, the gaze became more stable once again (Fig. 7-3) with hand coupling becoming less stable. Just prior to the onset of Fluency, hand-hand meanline spiked, marking the beginning of a new stability (Fig. 7-4) lasting throughout the Fluency stage (Fig. 7-5). Gaze meanline also moderately increased at this time (Fig. 7a-4).

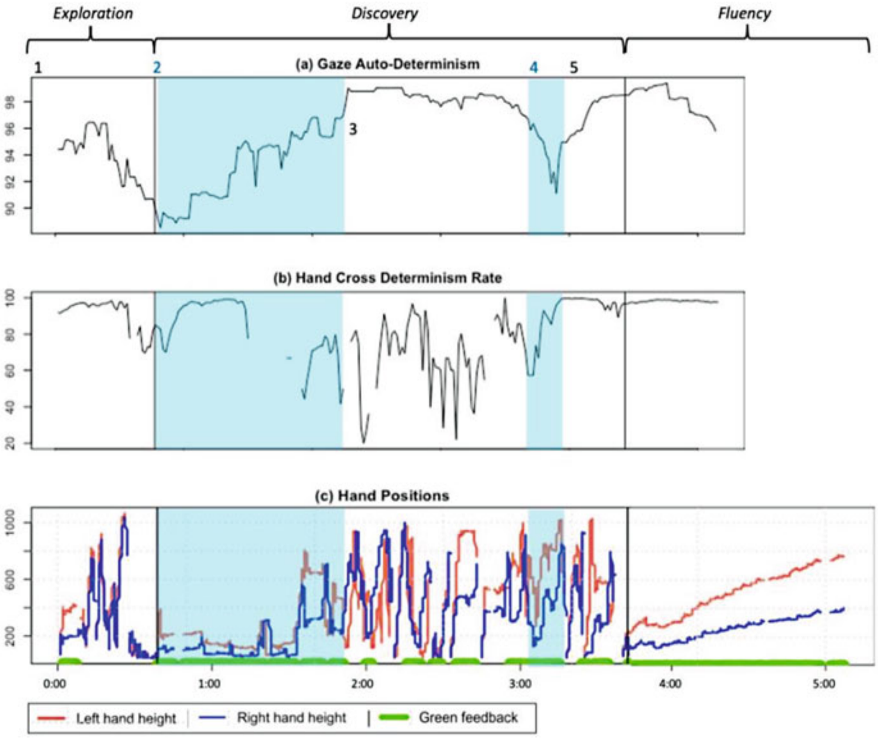

Fig. 6 Sliding window plots of hand and gaze percent determinism over time (window size = 50 seconds, step size = 1 second). Time series of hand positions included for reference. (**a**) Gaze determinism over time. Determinism alternately drops and increases throughout the Discovery stage and increases at the end of the Discovery stage. (**b**) Hand determinism over time. Determinism begins high in Exploration and early Discovery, becomes lower and more variable through mid-Discovery, and increases to a consistently high rate at the end of Discovery. (**c**) Plot of right-hand (blue line) and left-hand (red line) heights over time during the task. Green indicates a moment when the participant fulfilled the target ratio of heights between their hands. All three graphs share the same time x axis. Alternating blue and white shaded areas 1–5 highlights five qualitatively distinct portions of the time series for ease of reference in the text

Hand cRQA Summary Finn exhibited high levels of cross-hand recurrence rate, determinism, and meanline at the start and end of the time series, with lower levels in between (Figs. 5b, 6b and 7b), indicating a loss of coupling and consequent recoupling.

Gaze aRQA Summary In contrast to his hands, Finn's gaze showed initially low recurrence and determinism (Figs. 5a and 6a), increasing at the end of the time series. These results indicate that his eyes initially visited varied locations in variable ways, becoming more recurrent and stable in the final stage of the activity.

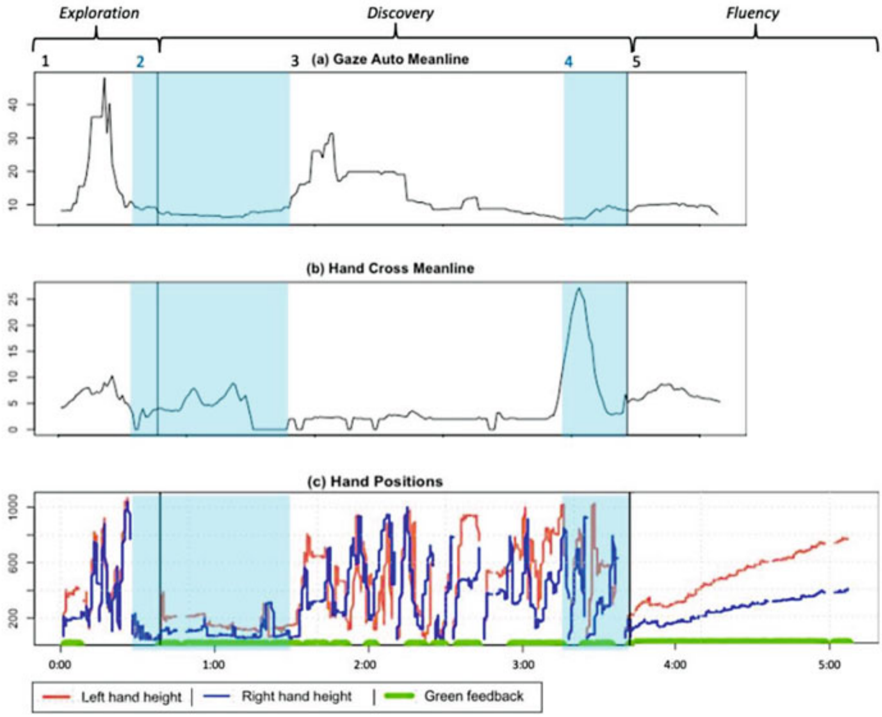

Fig. 7 Sliding window plots of hand and gaze meanline over time (window size = 50 seconds, step size = 1 second). Time series of hand positions included for reference. (**a**) Gaze meanline. Meanline starts high, drops at the onset of the Discovery stage, increases and decreases through mid-Discovery, and increases slightly at the end of Discovery leading into Fluency. (**b**) Hand meanline. Meanline starts relatively high, drops mid-Discovery stage, spikes sharply at the end of Discovery, and remains relatively high in Fluency. (**c**) Plot of right-hand (blue line) and left-hand (red line) heights over time during the task. Green indicates a moment when the participant fulfilled the target ratio of heights between their hands. All three graphs share the same time x axis. Alternating blue and white shaded areas 1–5 highlights five qualitatively distinct portions of the time series for ease of reference in the text

Intermodal Dynamics by Stage During the Exploration stage, gaze recurrence rate and determinism were low while hand recurrence rate, determinism, and meanline were high; although the hands were coupled with one another, eye movements were unstable. During the Discovery stage, Finn exhibited alternating gaze and hand dynamics: gaze determinism was high when bimanual determinism was low, and vice versa (Fig. 6a and b) and gaze meanline was high when bimanual meanline was low (Fig. 7a and b). At the onset of the Fluency stage, both hand and gaze synchronously stabilized and grew more predictable, exhibiting increased determinism.

4 Discussion

4.1 Interpretation of Findings

Each interactional modality measured in this case study exhibited distinct dynamic across the three task stages. Gaze dynamics, as measured by aRQA of fixations on areas of interest, began with highly variable behavior. This finding is consistent with the Gibsonian view that the function of exploratory movement is to expose new possibilities for action (J. Gibson, 1966), a process of information-detecting behavior that then supports perception of affordances for action attuned to local conditions (Adolph, 2019). Here, the gaze would offer new possibilities for action through exploration of a variety of locations relative to the hands.

In contrast to gaze, hand coordination, as measured with cRQA of right- and left-hand heights, de- and restabilized. The evolution from a predictably coupled, stable state through a less-stably-coupled and less-deterministic stage, through to another predictably coupled, stable state suggests a process of phase transition. Rather than a gradual increase, the participant's bimanual coordination changed suddenly, consistent with what Kostrubiec et al. (2012) term bifurcation. In contrast to shift, where a system maintains the same overall configuration, but with a slight pattern change, bifurcation is an abrupt reconfiguration. The right-hand–left-hand system shifted from one dynamically-stable coordination-pattern attractor to another, not unlike a horse breaking from a canter into a gallop. However, this transition included a lengthy transitional period of relative instability. The initially stable coordination destabilized when the participant began to elicit green feedback from the interface. We propose that finding green introduced a new constraint on the participant's movements, rendering the prior coordination incompatible and catalyzing the onset of a new attractor.

Looking intermodally across hand and gaze dynamics, each stage was characterized by distinct meta patterns: disconfluence of hand and gaze during Exploration, increasing confluence during Discovery, and high confluence during Fluency. Towards the end of the Discovery stage, a coordination of coordinations (Piaget, 1970) developed wherein the coordination between the left- and right hands became coordinated with newly developed gaze structures spanning different screen locations. This recalls Piaget's theory of reflective abstraction, whereby higher-order knowledge arises from observations on lower-order coordinated actions and reorganizing them into new coordinations (Piaget, 1970; see also Abrahamson et al., 2016). Here, higher-order perceptual structures emerge through coordination between modalities, which in turn informs coordination within each modality. The convergent increases in determinism across hands and gaze at the onset of the Fluency stage show an emergent meta-coordination that exhibited high predictability. The co-occurrence of increased gaze recurrence and increased hand coupling in this case study suggests a mutual influence of gaze pattern and hand–hand coupling. Only when the gaze pattern became more consistent did the hands recouple into a multiplicative pattern. The eyes became more deterministic and consistent only as the whole system solved the movement problem. The participant's solution was not simply multimodal, attained in each modality separately; it was intermodal, with hands informing gaze, and gaze informing hands.[4] Only through higher-order coordination did the participant fully achieve the task directive to move their hands smoothly in green. These results show that the phase transition arising in hand coordination fits within a larger dynamic. Whereas the initial hand–hand coupling was intramodal, the ultimate hand–hand–gaze coupling that emerged was intermodal. This suggests that learning to perceive the central ratio variable involved intermodal coordination of vision and kinesthesia. Consistent with dynamic system theory, the impact of hand and gaze changes is not additive; instead, we observe hands-informing-eyes-informing-hands iteratively, reciprocally, intermodally. The emergent movement is greater than the sum of its individual gaze and hand components, where each modality's pattern is impacted by the other, giving rise to a qualitatively new movement form of smooth multiplicative movement.

The process of specifying information detection that supports enactment of the target bimanual coordination pattern spans the kinesthetic and visual systems in this case study. Each motor skill has its own problem space characterized by particular affordance relations (J. Gibson, 1966). The MIT-P design has at its heart a Gibsonian higher-order variable (J. Gibson, 1966) of ratio, which participants learn to perceive through action. Although the MIT-P activity instantiates ratio in a situated context, ratio is conceptually scale-free, transcendent beyond specific spatial magnitudes. Following Gibson, we conjecture that the attentional anchor perceptual structure is in fact not a visual-specific structure, but a spatial–relational "amodal" one, perceivable through different modalities. Thus, whereas ratio's detection here is specified by information in the visual–kinesthetic array, it could also be instantiated using other modalities in the context of other instructional designs.

Some more recent ecological psychology work has challenged Gibson's view of multiple, overlapping perceptual systems, arguing instead for a single irreducible supermodal perceptual system (Stoffregen & Bardy, 2001; Stoffregen et al., 2017). In this view, direct perception is a characteristic of a global array made up of the irreducible superordinate structures across information available through the different sense organs. Here, perceptual systems function relationally and cannot be understood independently. Behavior simultaneously affects multiple ambient energy arrays, and relations between different ambient energy forms provide critical information about the animal–environment interaction. Supermodal structures exhibit emergent properties, more than the sum of their sensory system parts. In this view, learning involves perceptual–motor differentiation of relevant structures in a global array spanning multiple forms of ambient energy and determining which referents are relevant. For example, learning to somersault involves discovery and control of the relations between vestibular, biomechanical, and optical patterns of

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[4] Although not visual or kinesthetic, it is worth noting that the auditory modality also played a role here as the tutor offered prompts and encouragements that solicited changes in dynamics, most notably by encouraging the participant to try moving-in-green.

energy. From this view, embodied designs like the MIT-P create the conditions in the global array for the detection of supermodal structures. From this perspective, learning to move multiplicatively in the MIT-P would be construed as a process of perceptual–motor differentiation of emergent structures already in the global array through discovery and control of emergent relational properties across the visual and kinesthetic sensory organs. The onset of fluency would be the result of supermodal direct perception of irreducibly superordinate invariant structures.

4.2 Theoretical Implications

For educational research, the study creates an auspicious empirical platform for heady philosophical and theoretical deliberations over foundational metaphysical framings of the mathematical learning process. We welcome nuanced conversations with researchers following the various flavors of 4E cognition, including enactivism and ecological psychology (qv. Di Paolo et al., 2021), over explanatory models that best account for our empirical findings and how these accounts might enhance our educational design for diverse intersectionally marginalized students (Tancredi et al., in press).

A consistent focus of research remains the relation between enactment, perception, language, symbols, and concepts. How do the various 4E theories model the educational process by which students come to reason mathematically about their enactment? More specifically, what would be a 4E account for the semiotic micro-process by which symbolic artifacts take on enactive meanings? For partial accounts, readers are referred to our ongoing publications (e.g., Abrahamson et al., 2011, 2019, in press).

4.3 Methodological Implications

Our results suggest the traction of RQA on questions of intermodal coordination. Multimodal learning data, particularly in the context of embodied design, provides a rich context that engages both fundamental biological perception–action processes and cultural forms, from mathematical symbols to language. RQA's sensitivity to complex idiosyncratic dynamics at multiple, nested, sequentially dependent interacting timescales supports the evaluation, deepening, and elaboration of theory. RQA also provides a means to map the application of complex dynamical systems and coordination-dynamics constructs onto messy processes such as tablet-based mathematics learning.

4.4 Practical Implications

The present study supports the thematic conjecture of action-based embodied design (Abrahamson, 2014), namely that task- and environment-design can elicit novel, conceptually salient perceptuomotor dynamics. This supports the use of embodied design activities and design frameworks in STEM education. By improving theorization of how learning unfolds in technologically enhanced environments, MMLA including RQA can inform both future design and the facilitation of learning in these environments. Modeling implicit patterns of multimodal dynamics could possibly shed light on fundamental questions of epistemology, such as characterizing cognitive relations between perceptuomotor skill and conceptual knowledge. Additionally, MMLA can leverage multimodal data streams to better detect and characterize key moments in the learning process, enabling more responsive scaffolding from both the technological platform and teachers. An analytic instrument able to reliably measure students' real-time performance landmarks could eventually feed into the response algorithms of an artificially intelligent tutor or tutor support system offering timely feedback.

4.5 Limitations

This chapter presents a nascent foray into intermodality grounded in a first case study. Limitations of this case include that the participant exhibited lower levels of fluent movement-in-green at the end of this part of the interview than other participants. It is likely that gaze stability and the coupling between hands and gaze developed further in future stages of the interview beyond the scope of this analysis. These dynamics warrant examination in a greater number of participants. Further work within and beyond the MIT-P can deepen understanding of these issues through inter-participant analyses quantifying the relationship between gaze and hand dynamics. Additionally, our analyses of intermodality remain qualitative in this case study. In future analyses, MdRQA could provide further traction on patterns of shared states across multiple modalities of data.

4.6 Future Directions

Future directions for this work include cross-participant analyses of the intermodal dynamics in multimodal systems, work with participants with a greater diversity of sensorimotor profiles, and work expanding the unit of intermodal analysis to incorporate the tutor-tutee dyad spanning hand, gaze, language, and tutor movements. In turn, such analyses can feed back into the design of responsive embodied-interaction settings for mathematics education, and perhaps beyond.

One important line of inquiry is around the relation between intermodality and generalization of learning. In general, skill-learning processes furnish attentional anchors available to deploy in novel contexts. Per Bernstein, learning develops new capacities that can be deployed in situations that differ along multiple parameters (Bernstein, 1996). However, according to E. Gibson (1969), distinctive modality-specific properties like color can inhibit intermodal transfer. An open question is how qualities specific to the visual modality such as color might affect learners' capacity to generalize or extend the ratio concept. In this activity, the color green begins as a task objective, then becomes task feedback, eventually taking on further meaning as a conceptual placeholder in reference to the new type of relation discovered through the activity (Abrahamson et al., 2011). Later in the activity, as the multiplicative mathematical expression is introduced, green fades from its discursive role as redundant to broader mathematical inquiry. Research on perceptual learning suggests that differences in learning pathway impact the adaptability of a coordination pattern attractor to novel situations (Yamamoto et al., 2020). Generalization of learning-through-the-MIT-P is a theory-generative direction for further empirical work.

To deepen our investigation of modality in movement-based learning, it is fruitful to explore other modal presentations of the MIT-P activity, such as versions using haptic or auditory feedback (see for example, Abrahamson et al., 2019; PhET Interactive Simulations, 2021; Tancredi et al., in press). These designs create the conditions for learners with a greater range of sensory profiles to engage with the activity. The learning trajectories of diverse populations who access activities through different modalities can shed light on amodal and intermodal dimensions of learning. For example, congenitally blind individuals have been found to have differently organized occipital lobe processing, processing spatial information using brain areas that respond to visual input in sighted individuals (e.g., Burton, 2003). In addition to kinesthetic, proprioceptive, and tactile information, auditory information is also used spatially. Studying blind learners' interactions with audio-haptic versions of the MIT-P can investigate intermodal equivalence in pathways to achieve motor skill. How might the dynamics of the activity in these versions compare to the color-feedback visual–kinesthetic version, for sighted versus congenitally blind learners, acknowledging the different cognitive architectures arising from different lifelong modal experiences? Additionally, research on pairs or groups of students with different sensory profiles, such as blind and sighted learners, can provide a rich context for studying the nature of shared mathematical meaning-making beyond traditional modal configurations.

5 Conclusion

We observed a case study in which a student progressed through exploration, discovery, and fluency in an embodied design context. Reaching fluency in this case consisted of achieving intermodal coordination: the stabilization of new gaze patterns together with smooth bimanual movement. These analyses shed light on the process of perception-for-action by which perceptual structures emerge through and for coordinating bimanual actions. Exploratory movements functioned to generate possibilities for action, leading to discovery of new intermodal, if not supermodal, affordances for action not available to the independent modalities individually. In this environment, the learner assembled his body into a tool for solving the problem, coordinating hands and gaze into one dynamically stable assemblage. These results suggest that multimodal data streams are more than mere entry points into a cognitive phenomenon, and are actively interacting, mutually influencing components of a larger dynamical system. Beyond multimodality, these data reveal conceptual learning as intermodal coordination. Rather than a centrally controlling, modality-agnostic concept nurturing from each modality, a parsimonious explanation for our results is that what we call learning is constituted by the self-organized coordination of different modalities. Interacting with the embodied design environment effectively cultivated the perception of an intermodally-invariant, relational structure through the soft-assembly (Richardson & Chemero, 2014) of a temporary hand–gaze coalition into a coupled relation instantiating multiplicative reasoning. Our findings point to the importance of MMLA work that attunes to intermodal dynamics of learning, both as a pragmatic resource for identifying key moments in learning and as a resource for refining theoretical understandings of learning processes.

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