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Balance Board Math: Exploring the Sense of Balance as a Basis for Functions and Graphing and Number Line Concepts

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Abstract

Can math concepts be experienced through the sensory modality of balance? Balance Board Math (BBM) is a set of pedagogical math activities designed to instantiate mathematical concepts through stimulation to the vestibular sense: an organ in the inner ear that detects our bodily balance and orientation. BBM establishes the different ways children spontaneously rock and move as the basis for inclusively exploring mathematical concepts together across diverse sensory profiles. I describe two activity sets where students explore focal concepts by shifting their balance on rockable balance boards: "the Balance Number Line," using analog materials to foster understandings of the number line and negative numbers, and "Balance Graphing," using sensors and a digital display to foster exploration of functions and graphing concepts, including the parameters of trigonometric functions and function addition. I outline proposed ways that engaging with concepts through balance-activating movement can change learners' mathematical thinking and learning.

Keywords Balance · Embodied cognition · Sensory · Inclusive design · Functions and graphing · Number line

Introduction

You have seen sinusoids, but have you felt one? The concept of balance gets evoked regularly in mathematics to teach concepts such as balancing equations, yet rarely do we engage learners' bodily sense of balance in the classroom. What if we learned concepts not just through seeing diagrams, touching manipulatives, or hearing language, but also through experiencing different balance sensations? How might our mathematical thinking change? What new avenues for inclusive learning might balance-based thinking open up for learners with diverse sensory experiences and needs?

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Balance Board Math (BBM) explores the vestibular (balance and orientation) sensory system as a basis for mathematical conceptualization within several conceptual domains. For example, learners use rockable wooden boards to experience the frequency of a sinusoidal function as a rate of rocking their bodies (Fig. 1), or a negative number as the counterbalance to its positive counterpart that brings their body back into balance. This snapshot describes how using the sense of balance to explore mathematical concepts, as enabled by digital technology, can prospectively change how learners enter into and think about mathematical concepts. BBM is inspired by the intersection of two design problems: one sensory and one cognitive.

Sensory Accessibility

People, especially children, often spontaneously fidget and move in ways that stimulate their balance sensory system: rocking, jumping, pacing, and twisting. Research on sensory processing and integration suggests that such movements are part of *sensory regulation*: seeking out optimal levels of sensory stimulation to control one's affect and arousal levels (e.g., Dahl Reeves, 2001). We crave different intensities of stimulation based on their neurology, with many neurodivergent individuals, including those on the autism spectrum and those with ADHD, especially likely to fall at extreme ends of the sensation-seeking spectrum (Dunn, 1997). For example, some people may need a high level of stimulation to the vestibular sense of balance and pursue this by rocking, bouncing, or pacing. Even if learners' sensory stimulation needs are identified and accommodated, these tend to only be addressed through offering learners peripheral sensory tools such as fidget toys. Pedagogical activities themselves often do not offer the sensory stimulation people crave and may even be incompatible with people's own spontaneous movements. *What if instead, we made people's spontaneous movements such as rocking their body part of mathematics instruction itself?* BBM seeks to offer learners differentiated vestibular stimulation as fundamentally integrated into instructional activities. In so doing, the motivation is to render math instruction more accessible and engaging to all learners, especially

Fig. 1 A pair of learners exploring Balance Board Math activities

those with higher needs for vestibular stimulation not met by current practice. Furthermore, the sense of balance offers an untapped shared resource across learners with different sensory profiles from each other, such as blind and sighted learners, to engage in learning experiences together.

Embodied Cognition and Instructional Design

In recent decades, a novel paradigm in cognitive sciences has emerged that highlights the centrality of bodily experience to cognition: *embodied cognition*. In contrast to computational models of cognition that characterize thinking as an amodal process of input, processing, and output, embodied cognition suggests that cognition evolved for and arises through multimodal experiences through our bodies with the environments. This perspective sheds new light on mathematical thinking and knowing, highlighting the multimodal nature of concepts, wherein even apparently abstract processes, such as solving algebraic equations, make use of our sensorimotor experiences of manipulating objects in the world (i.e., Hutto et al., 2015). Empirical work in education inspired by this perspective has found bodily activity to influence and express thought, from gesture to background movements (for a summary, see, for example, Shapiro & Stolz, 2019). One particular strand of embodied cognition, enactivism, highlights how cognitive structures emerge from patterns of perceptually guided action in the world (Varela et al., 1991). *Action-based embodied design* concretizes this theoretical perspective, seeking to foster new perceptuomotor coordinations as proto-instantiations of mathematical concepts (Abrahamson, 2014; Alberto et al., 2022). In this view, teaching can be made more effective by explicitly cultivating sensorimotor activity as the core of conceptual thinking.

The Unsung Sense of Balance

The notion of balance has long been leveraged in mathematics to support reasoning about mathematical equations (i.e., MacGregor, 1991). Furthermore, research on the bodily sense of balance (the *vestibular system*) reveals its importance to cognitive development, spatial reasoning, and numeracy (e.g., Mast et al., 2014). Despite the vestibular sense's importance to foundational math competencies, it is not intentionally stimulated by extant math pedagogical designs. Could explicitly engaging this sense support math skills and conceptual learning?

To date, balance-based conceptualizations have been explored in the context of abstract reasoning about social justice issues, wherein it was found that using a balance-driven input device rather than a handheld one to make comparisons led participants to be more deeply affected by injustice (Antle et al., 2013). Balance-based input devices have also been explored in physics learning about seesaws (Pouw et al., 2016). BBM represents a first foray into vestibular-driven conceptualizations of mathematical concepts. BBM seeks to pose sensorimotor challenges wherein learners figure out new patterns of balance and disbalance, drawing upon mathematical tools such as graphical representations and number lines to refine their control and explanation of these patterns.

The BBM Design

BBM is driven by the conjecture that integrating sensory-regulatory and cognitive affordances of movement can render conceptual learning more accessible. The design principles guiding BBM (Tancredi et al., 2022b) are as follows:

1. Foster movements that ground mathematical concepts in experiences of balance. This entails the congruence of balance-activating movements with focal mathematical concepts.
2. Support learners' discovery and control of dynamical properties. Learners discover feedback from the digital platform and/or their sensory experiences that guides them to explore, refine, and think about their movements.
3. Invite embodied self-regulation during and through instructional activities. Learners are welcomed to move their bodies in ways commonly used for sensory regulation.
4. Inclusively adapt to different sensory profiles. The design is built to enable learners to rock at any intensity to accommodate a wide range of sensory stimulation preferences.

For more detail on the design and rationale of BBM, see Tancredi et al. (2022b).

The aim of the present snapshot is to illustrate how balance-based approaches create opportunities for new ways of conceptualizing math, drawing upon brief examples from multiple studies wherein BBM has been tested with hundreds of children and adults in semi-structured interviews, classrooms, and informal learning settings. These include a Balance Number Line case study with a middle schooler on the autism spectrum (Tancredi, 2020), semi-structured interviews with elementary and middle schoolers with a range of sensory profiles (Tancredi et al., 2022b), and a classroom study with four high school classes using the Function Addition activity (Tancredi et al., 2023). Sensation-seeking neurodivergent learners, including children on the autism spectrum, have been focal participants throughout the project to examine how the design interacts with their sensory needs and practices.

To illustrate the pedagogical potential of balance-driven math experiences, I will describe two BBM configurations and some mathematical explorations possible with each: the Balance Number Line and Balance Graphing. Both of these configurations by default use a balance board with a flat surface and semicircular prism base, rocked from a seated position (i.e., Fig. 1) (see Appendix 2 for a discussion of other prospective balance board experiences). The board is rocked left-to-right for Balance Number Line and front-to-back or left-to-right for Balance Graphing.

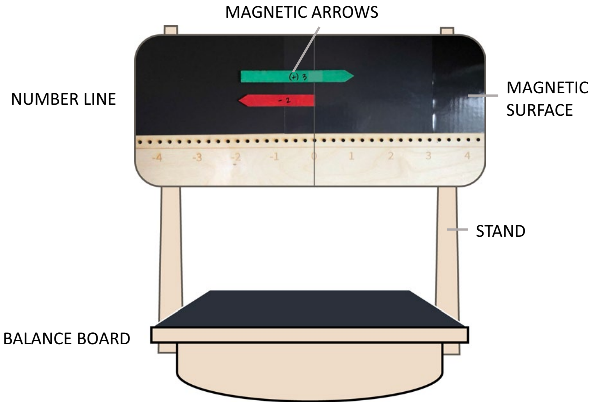
The Balance Number Line

A first set of BBM activities, the Balance Number Line (Table 1), establishes balance-based conceptualizations of absolute value and negative numbers. Learners sit on a balance board and slide their hands along a number line in front of them (Fig. 2). Zero

Table 1 Balance Number Line activities

Activity	Purpose	Balance experience
Move-in-balance	Provide a balance-based definition of negative numbers	Learners experience positive numbers and their negative counterparts as perfectly balancing one another on a continuous spectrum by moving their hands symmetrically outwards from 0
Expressions and equations	Compare different equations and their solutions, differentiating between sign and operation	Learners experience arithmetic as shifting states of balance elicited by sliding their hands along a number line, and solutions as a specific final angle of tilt

Fig. 2 The Balance Number Line



on the number line is aligned with the middle of the balance board and the child's body. When one moves their hands, the shift in weight yields a resulting tilt in the balance board. For example, moving one's hand to − 2 results in a slight lean to the left, whereas 5 results in a large lean to the right. This also applies when the two hands are placed at different points on the number line: − 3 and 3 are experienced as being in balance, whereas − 3 and 4 yield a slight lean to the right. Black magnetic sheet coats the upper half of the board to allow for affixing magnetic arrows that can be used to keep track of a sequence of movements. I will discuss some Balance Number Line activities in more detail in the "Concepts Beyond Vision" section, as well as in Appendix 1.

Balance Graphing

A second set of BBM activities, Balance Graphing (BG), establishes balance-based conceptualizations of functions and graphing concepts. BG joins other dynamic graphing activities (Duijzer et al., 2019) that use sensors to enable learners to generate their own graphical representations in real-time. In BG, users create graphs on a digital display by rocking a balance board (Fig. 3). The display varies by activity, always offering graphical representations of the board angle. The board's angle, detected by an inclinometer sensor affixed to the board, is relayed via Arduino to a computer program and graphed (Fig. 3). After a graph is drawn, it remains visible on the screen until a new graphing round is initiated.

BG activities (Table 2) annotate or augment user's graphs with feedback to foreground different mathematical properties. This includes real-time feedback from the digital display as they are graphing, and summary feedback at the end of each graphing round. Feedback information is conveyed through visual features and sound (*sonification*). For example, in one activity, Function Addition (described further in the "Coordinated Action" section), the movement of each board generates one of the colored graphs, and the sum of these two functions is displayed in black. Figure 3 shows one such graph. Here, the left balance board (blue) tilts to the right, and the right board (orange) tilts the same degree left. Holding these positions, they trace lines at $y = -1$ and $y = 1$, respectively. A black line shows the sum of these two graphs, here roughly tracing $y = 0$.

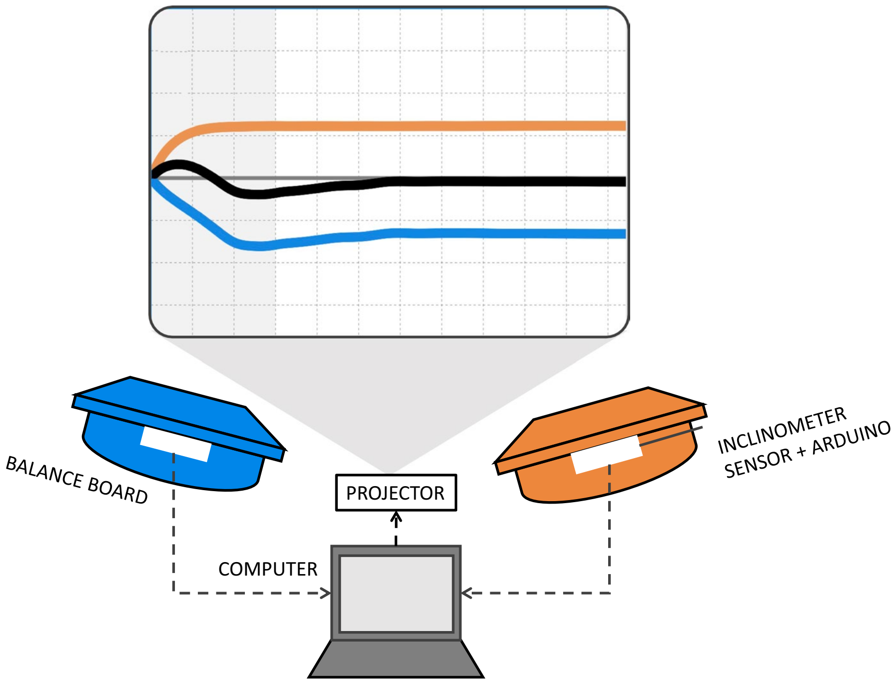


Fig. 3 Balance Graphing configuration

As we will see in later examples, BBM activities can be used alone, in pairs, or in teams with one or two balance boards.

BG activities (Table 2) target specific areas of trigonometry highlighted as challenging for students by teacher partners. These include the parameters of trigonometric functions, including understanding the relationship between frequency and period, as well as the symmetrical nature of amplitude, the correspondence between the sine and cosine functions, and the unit circle. BBM uses experiences of balance and discovery-based learning to foster understanding of these concepts. For example, in the BG Amplitude Exploration activity, learners experience the symmetrical nature of amplitude as rocking to the same angle left and right. In the BG Frequency Exploration activity, learners can relate their felt sense of rocking speed to how many periods they generate on the screen in a given time period. In the BG Unit Circle activity, learners reconstruct the unit circle by modulating sine and cosine. The exploratory and movement-based nature of these activities offers new ways into these challenge areas. See the supplementary video for some examples of learners exploring BG activities. More detail on specific BG activities are included in Appendix 1 (Function Exploration) and the forthcoming sections (all other BG activities).

As context for all BG activities, I will describe here the precise way inclinometer data is translated to graphical output. A grid is visible on the display, without labels for x and y. The x-axis corresponds to time (in seconds); graphs are generated at a constant rate. The y-axis corresponds to board angle (in degrees of tilt). The *angle* of the board's inclinometer sensor is translated to specific *y values* for the graph. Angles on one side of flat correspond to positive y values and the other, to negative

Table 2 Balance Graphing activities

Activity	Purpose	Balance experience	Key display features
Function Exploration	Experience "being the graph"	A graph's slope and contour is experienced by rocking as if surfing or riding over the surface of the graph	Color, sound, and percentage feedback express how closely they have followed the target graph
Amplitude Exploration	Identify and control the amplitude parameter of trigonometric functions (as a symmetrical phenomenon and as distinct from frequency)	The amplitude of a function is experienced as the degree of tilt in either direction and can be analyzed in relation to the y-coordinates of graphical minima and maxima	Real-time stars (appearing at local minima and maxima that meet target amplitude) and a summary count express how consistently they have rocked at a focal amplitude
Frequency Exploration	Identify and control the frequency parameter of trigonometric functions; perceive one period as a unit and relate period to frequency	Frequency is experienced as rocking rate and can be analyzed in relation to the observed period length of a graph	Color and sound feedback convey how closely they matched a focal frequency for each graphical period
Function Addition	Develop graphical intuition around how the sum of two functions behaves (including when functions are on the same or opposite sides of the x-axis)	Function addition is experienced as the moment-to-moment relationship between one's own board's angle relative to a peer's	Each player's graph and the sum of their graphs appear in real-time as they rock
Unit Circle	Build understanding of the relationship between the unit circle and the sine and cosine graphs	Sine and cosine are experienced as a distinct rocking coordination pattern of alternating movement and varied rate of change that enable tracing the perimeter of the unit circle	Sine player controls x and cosine player controls y, like an etch-a-sketch. Each player's graph, as well as their joint location on the coordinate plane, are displayed simultaneously

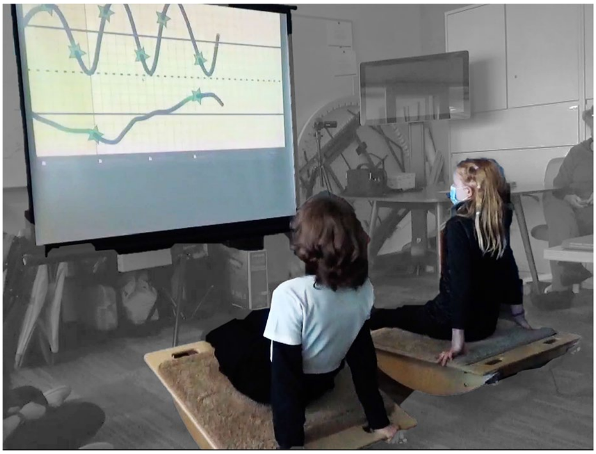


Fig. 4 A round of Amplitude Exploration in progress

values. For example, when rocking front-to-back ("roller coaster" style), lifting the front of the board by leaning backwards generates positive values, and lowering the front of the board by leaning forward, negative values. When rocking left-to-right ("surfing" style), the right edge of the board is treated as the front since it faces the direction the graph progresses. Thus, to move one's graph upwards into the positives, one creates an "uphill" angle with the board, and to move it downwards into the negatives, one creates a "downhill" angle. Holding the board in a given position yields a constant *y* value. (See Appendix 3 for a discussion of alternate mappings).

To accommodate learners' rocking preferences and compatibility with different displays, graphing rate and sensor sensitivity are adjustable to adapt the graph's scale and rate of appearance to a child's preferred range and speed of rocking, with default units being about 10° for *y* and about 2 s for *x*.

Example Learning Trajectory with BBM

BBM offers a digital environment that, through feedback, invites learners to move in new ways, such as rocking at a given amplitude. Let us track the thinking of a pair of middle school girls, Molly and Annette, as they generate a series of graphs with BBM in the Amplitude Exploration activity.

The BBM Amplitude Exploration activity (Fig. 4) is designed to foster attention to the amplitude parameter of sinusoidal functions. Learners are invited to "find green stars," which appear when they rock at a particular (modifiable) amplitude.[1] Finding stars below the *x*-axis generates a low audio tone, and above

[1] This activity presently focuses on sinusoidal functions centered on the *x*-axis because having the function centered at 0 (board in balance) facilitates perception of symmetry in rocking to the same degree to either side. However, in this activity, I have observed learners to also explore generating sinusoids centered on one side of the *x*-axis, suggesting that with some modifications, this activity might also offer means to explore sinusoids centered elsewhere.

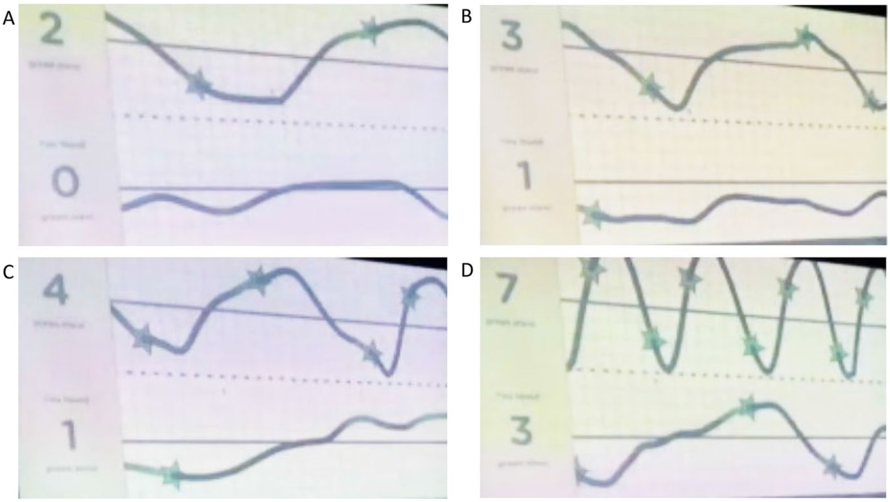


Fig. 5 Sequence of Amplitude Exploration graphs generated by Molly and Annette

x, a higher one. Through rounds of graphing and reflection, learners come to recognize and modulate the amplitude of their graphs by symmetrically controlling the angle of their rocking.

When first introduced to the Amplitude Exploration activity, Molly and Annette are invited to "find as many green stars as possible" and informed that they can try as many times as they would like. In the first round, Annette (Fig. 5a, upper portion of screen) finds two green stars, one below the *x*-axis, and one above. Molly mostly graphs below the *x*-axis (Fig. 5a). Towards the end of the round, her graphing line begins to turn green. She does not find any stars during the round. Invited to reflect on their graphs, Annette comments: "Hmmm. I see, it's like that" and gestures a wave form with her arm. When prompted to explain further, she says that to get green stars, one must use a "wave pattern." She asks the researcher if she is allowed to know how many stars there are in total, a question the researcher invites her to think about during subsequent rounds. Molly, meanwhile, comments that her graph was "almost there," since her line turned green but a star did not appear. Molly and Annette share that next round, they plan to "do the wave thing again" (Annette) and "try [to] follow the green line" (Molly), i.e., to continue rocking in the same direction when the line she generates starts turning green.

The pair completes another round of graphing (Fig. 5b). This time, Annette finds three stars, and Molly finds one. Annette gestures the sinusoidal shape of her graph with her hand and arm again and shares, "I feel like there should be more stars, like I feel like, maybe it should go like up-down-up-down faster?" When asked to describe where the stars are that she found, she notices their y-coordinates: "oh yeah! Maybe it's like, 'cause they are- all of them were second to bottom, about second to bottom, second to top square." Molly, meanwhile, notes that while she also got the first star, the star's locations for each of them are "not in the same exact spots"- that is, she notes the flexibility of the stars' x-coordinates. Thus, through exploration, Molly and

Annette have arrived at a proto-notion of amplitude as a symmetrical feature of a sinusoidal function, specified by y-coordinate rather than x-coordinate.

In their next round (Fig. 5c), Annette completes two cycles in the graphing time period, resulting in four stars. Molly finds a first star at a local minimum and almost reaches a second star, but is slightly shy, her line turning green. They try graphing again immediately. Finally, Annette finds seven stars, and Molly finds three (Fig. 5d). Annette comments: “I guess mine is like very high frequency, or, kind of high frequency,” noting she achieved this by “mov[ing] more up and down so it made shorter waves.” Molly commented on how she “kept going in the same direction” when the line started turning green to achieve the stars. The researcher then repeated Annette’s question from earlier: how many stars could one get in this activity? Annette initially expresses that since the stars are evenly spaced, perhaps she has reached the maximal number, but Molly reasons, “I think if you go faster up and down, then maybe there’s more.” Annette agrees, noting, “maybe if you go like double as fast then you will find almost double the amount of stars.” Thus, Molly and Annette have begun to parameterize their graphical creations according to both amplitude and frequency. In the next section, we will look more closely at how using BBM transforms learners’ mathematical thinking.

Transforming Math Knowledge

BBM transforms mathematical thinking and knowing in several ways:

1. Concepts beyond vision: restructurating concepts through the balance sensory modality
2. Being-the-graph: cultivating whole-body experiences of disciplinary forms
3. Coordinated action: coordination dynamics as distributed thinking
4. Embodied design: disciplinary tools as resources for action

I elaborate each below using an example BBM activity and how learners have interacted, or can interact, with that activity.

Concepts Beyond Vision: Restructurating Concepts Through the Balance Sensory Modality

Wilensky and Papert (2010) coined the term *restructurations* to describe a shift in the encoding of knowledge in a domain based on a shift in representational infrastructure, for example, a shift from Roman numerals to Hindu-Arabic numerals. Restructurations can be evaluated in terms of how they expand access to and capacity within a discipline, including by accommodating diverse learners, enhancing engagement, and enhancing fit with human evolutionary-cognitive dispositions.

BBM restructurates mathematical thinking through the balance sensory modality, rendering concepts explorable and expressible using experiences of balance and tilt. For example, in the BBM Balance Number Line activities, negative numbers’

Fig. 6 Moving-in-balance, solved by a middle school student (see the supplementary gif file to see the solution in action)

opposition to their positive counterparts is conceptualized as counterbalance. Learners establish this conceptualization through different exploratory activities on the board. One such activity is the “move-in-balance” challenge. A learner is asked, “do you think you can find a way to move both hands simultaneously along the number line while maintaining the board in consistent balance?” The solution to this problem entails moving the hands apart symmetrically such that they are always equidistant from 0 (for example, passing through − 1 and 1 simultaneously) (Fig. 6). Through problems like this one, learners come to conceptualize a number and its opposite as in balance with each other. For example, a middle school student on the autism spectrum experienced placing his left hand on − 1 and his right on 5 as tilting him “way to the right,” whereas − 2 and 2 felt “zen.” Based on these two experiences, he spontaneously generated and tested the prediction that − 5 and 5 would also feel “zen.” These activities offer a view of negative and positive numbers as not just equidistant from 0, as is visible on a visual number line representation, but as bearing mutually neutralizing displacements in opposing directions.

Being-the-Graph: Cultivating Whole-Body Experiences of Math Representational Forms

Research on gesture suggests that those with greater graphing expertise show differences when asked to represent graphs gesturally: they tend to gesture as if “being the graph” (using more dynamic, whole-body movements, even taking a perspective of being inside the graph) rather than merely “seeing the graph” (gesturing as if sketching the graph on paper) (Gerofsky, 2011). The sense of balance offers a prospective means to realize a “being-the-graph” experience by generating a graph’s contours with their body. Using a balance board to graph, generating a high-slope graph feels steep, a high-frequency graph feels rapid. Learners can imbue the graphical representation with meanings from these embodied experiences.

Let us take the example of the BG “Frequency Exploration” activity, designed to orient learners to sinusoidal graphs’ period and frequency. This activity links felt experiences of symmetrical rocking to periods on the graph and rocking speed

Fig. 7 A round of Frequency Exploration in progress

to frequency. As they rock, each local maximum and minimum and each passage through the x-axis generate a sound: low for the minimum, medium for the x-axis, and high for the maximum. Each time a child completes one period (rocking in each direction and back to center), they receive color and sound feedback reflecting their proximity to a target frequency (a modulable parameter). The background behind each period on their graph turns green for matching the target frequency, yellow for almost matching, and red for being farther off.[2] For example, in Fig. 7, the child on the left has generated three different periods in the upper graph. Based on proximity to the focal frequency, each has turned a different color: red, yellow, and green. They also hear one of three chords: highly consonant for green, somewhat consonant for yellow, and dissonant for red.

To illustrate, middle-schooler Kyle was invited to explore Frequency Exploration and "try to turn the whole screen green" (see Tancredi et al., 2022b). Across rounds of exploration, Kyle came to connect the contours of graphs, including the graphical period, with his felt sense of rocking speed. He recognized an inverse relation between the number of periods ("hills") drawn and their width: "maybe if they're like, the hills are further apart [they will be green]- but then that doesn't make as many hills." He also related his felt experience of rocking speed to the number of periods drawn in a given timespan: "I went a lot slower, and I only got three cy- hills, like cycles."

Kyle and other children's gestures showed increased hallmarks of "being-the-graph" in post-interviews after BBM. Compared to a pre-interview, when asked to gesturally show a set of sinusoidal graphs, the children used larger scale, more dynamic gestures (Tancredi et al., 2022b). For example, Kyle leaned left and right to different angles reflecting graphical amplitudes (Fig. 8) whereas his torso remained static in the pre-interview. He also used language related to speed, such as "slower, but rocking a lot farther." Thus, even static, 2D representations of graphs appear to evoke multimodal experiences for children through BBM.

[2] The sensitivity for these thresholds can be adjusted.

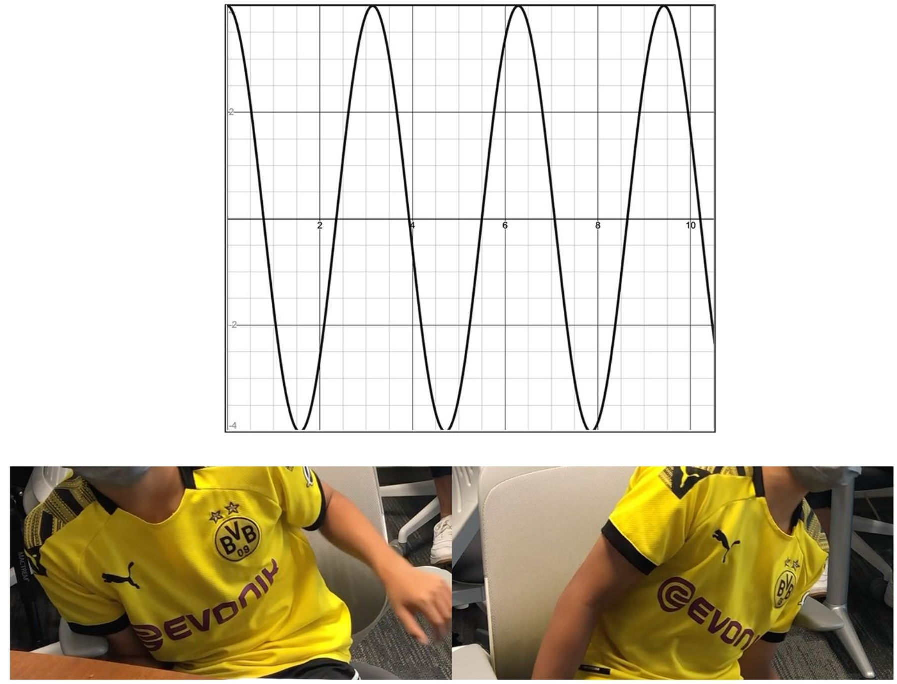


Fig. 8 Static graph and a child's gestures when describing it after using BBM

Coordinated Action: Coordination Dynamics as Distributed Thinking

Through multi-player interactions, BBM players can experience the relationship between two quantities as the relation between their own movements and that of a peer. For example, in the BBM Function Addition activity, two boards generate graphs on the same graphing area, each with a different color (blue or orange), and the sum of their two graphs is shown in black in real-time (Fig. 8). Function Addition is designed to build learners' graphical intuitions around the meaning of adding functions, including how a sum function relates to its component functions, how the sum of two functions relates to their value at any given x, and how a given sum function can be achieved with different combinations of functions. To achieve desired functions, learners must attend to the coordination of their joint activity, attending to features such as frequency, amplitude, and slope.

When invited to explore this activity at a science fair, even pre-primary school children frequently explored rocking in opposite directions to bring the sum line towards the x-axis and rocking in the same direction to drive it to the extremes of the screen.

Let us follow a class of high school students as they explore Function Addition (Tancredi et al., 2023). Students began by exploring a few rounds of graphing to observe the relationship between the players' graphs and the black graph. Next, they were given a focal function such $y = 0$ (tracing the center line), a high-amplitude

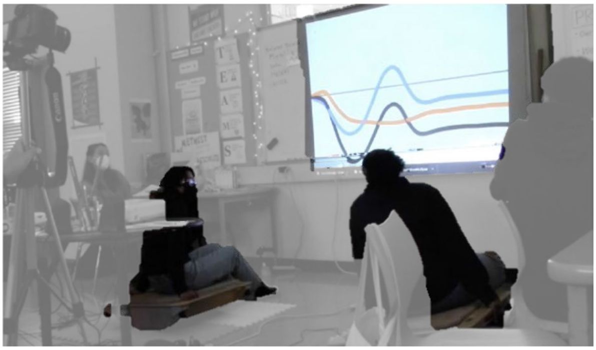


Fig. 9 Two high school students explore Function Addition

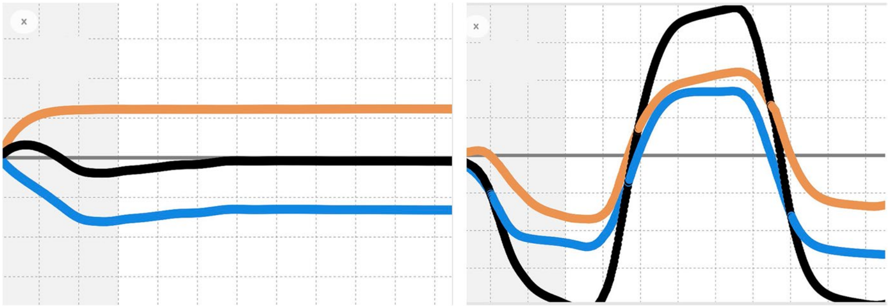


Fig. 10 Two examples of student-generated Function Addition graphs. **a** Goal: sum function $y = 0$. **b** Goal: a high amplitude sinusoidal sum function

sinusoid such as $y = -\left(\frac{5\pi}{6}\right)\sin(x)$ (a wave that reaches the very top and very bottom of the screen), a line with a specific slope such as $y = -2$, and complex functions such as $y = \sin(x) + x$ and invited to brainstorm and sketch ideas to generate these sum graphs, taking turns testing out their ideas on the boards in front of the class (Fig. 9).

Student teams generated multiple solutions to each challenge. For example, one student solution to generate a sum function of $y = 0$ (Fig. 10a) was for orange and blue to start at $y = 0$, then have orange increase to $y = 1$ while blue simultaneously decreases to $y = -1$, holding these positions. Other student solutions to this challenge included both staying perfectly still and drawing opposite phase sinusoids. An example student solution to generate a high amplitude sinusoid was for both blue and orange to draw sinusoids that were in phase with one another at a shared frequency and amplitude of about 2 (Fig. 10b).

As they tested and refined solution ideas, the students began by focusing on their direction of movement, attending with increasing specificity to their angles, and eventually the boards' relative positions to one another. For example, one group initially planned by pointing in a direction of travel ("let's go this way first"). After a couple more rounds, they started to use counting aloud ("one, two, three" on each side) to synchronize their rocking. Next, they attended to the

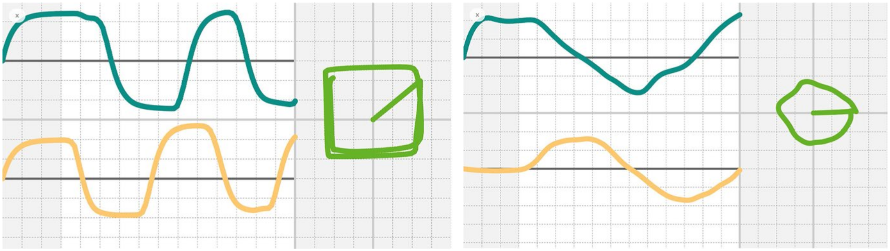


Fig. 11 Screenshots of drawings using sine and cosine. **a** Drawing a square. **b** Drawing a circle

specific angles of the boards (one tilted "all the way left," the other "tilt[ed] all the way to the right"), even rehearsing these angles prior to graphing. Function Addition enabled different bodily coordinations in time and space to become the basis for understanding the relation between two complex quantities. The class showed improvements on pencil-and-paper function addition problems following these explorations.

The BBM Unit Circle activity provides another example of coordinated action as a basis for conceptualization. Unit Circle is designed to support learner's understanding of the relationship between the unit circle and the sine and cosine functions. One board controls sine, and the other controls cosine. In addition to displaying the graphs of their rocking, as in other activities (sine in teal on the upper left, cosine in yellow on the lower left), Unit Circle displays a second visualization: a trace of their position in a cartesian grid with sine controlling *y* and cosine controlling *x*, analogous to an etch-a-sketch (Fig. 11, shown in green on the right).

One pedagogical trajectory with Unit Circle, applicable for both those somewhat familiar and wholly unfamiliar with trigonometry, is to first invite users to generate horizontal or vertical lines (wherein one board moves and the other stays still), followed by a diagonal line (requiring simultaneous, coordinated movement). Subsequently, learners are invited to generate a square, requiring timed alternation of movement. From here, they are invited to try to trace a circle. This requires a process of smoothing the functions they made to generate a square, slowing down through the peaks and troughs while maintaining the same phase relation with one another. When perfected, the users will have drawn the sine and cosine functions. The activity is designed such that the canonical forms of sine and cosine constitute the enacted solution to the presented movement problem. Learners experience the phase relation of sine and cosine, and the relationship between each point along these functions and each point on the unit circle, through their continuous coordinated action. Thus, learners come to know these functions' form and phase relations, as they have reinvented them to trace the perimeter of the unit circle. Learners come to understand these representations through coordinated action.

In these activities, learners are distributed, each controlling one of interacting variables in a system. In so doing, they can leverage core human competencies of concerted action (Melser, 2004) to instantiate and understand the relations among components through coordinated joint action.

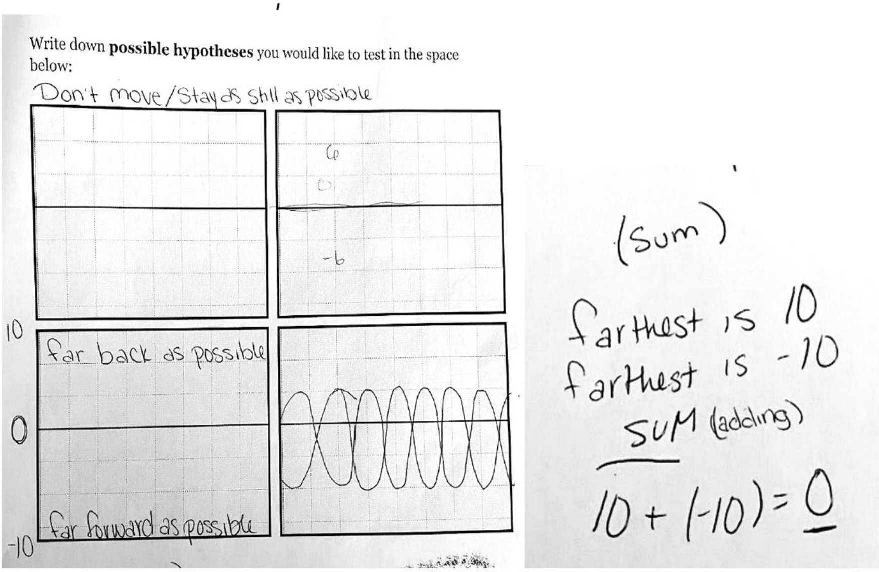


Fig. 12 Students' written brainstorming to generate a sum graph of $y=0$

Embodied Design: Disciplinary Tools as Resources for Action

In the embodied design framework (Abrahamson, 2014) for enactivist pedagogical design, learners explore a *field of promoted action* (Reed & Bril, 1996) wherein through feedback, they figure out how to move in a new way to think in a new way. Disciplinary tools and artifacts are introduced as resources for controlling and communicating about actions, prompting reflection and discourse that establishes these new ways of moving as reflecting disciplinary concepts.

BBM instantiates this design approach; grids, number lines, and numbers become resources for enacting and communicating about ways of moving: for example in Frequency Exploration, learners might discuss how many units across correspond to green and calculate how many total greens are possible.

In a high school classroom exploration of Function Addition, learners progressed from qualitative thinking ("[rock] slower") towards more discrete and quantitative thinking ("go 2 down [on the grid])." The learners began to quantify to reason about and plan their movements, for example jotting down "$10+(-10)=0$" (Fig. 12). BBM thus instantiates the design principles of action-based embodied design (Abrahamson, 2014; Alberto et al., 2022) by rendering mathematical disciplinary tools as resources for refining their enactment of solutions to motor control problems.

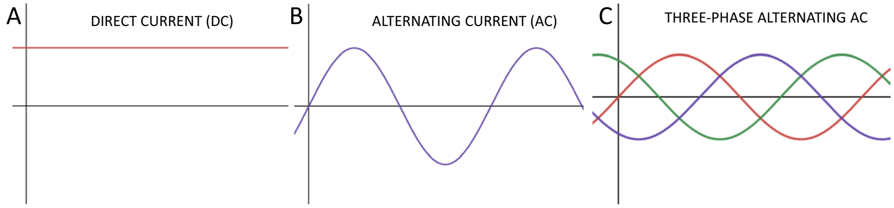


Fig. 13 Types of current

Discussion

The concepts explored here reflect the potential of technology to foster balance-activating movements as a resource for thinking about a range of math concepts at different levels. Beyond the concepts discussed here, balance-based models could shed light on other areas within and beyond mathematics. For example, the board offers a whole-body experience of slope, of interest for relating a graph to its derivative (Appendix 3). In physics, Balance Graphing activities could be developed to contrast voltage and current in alternating current (in which voltage switches periodically in a sinusoidal function) and direct current (in which voltage maintains a constant direction), as well as relations among different signals in contexts such as the three-phase alternating current waves sent out by power plants (Fig. 13). Multi-board configurations also enable embodied understandings of coordinated action, which may be of interest for activities exploring the coordination in biological systems, such as that of the contractions of valves of the heart.

The presented design uses a ubiquitous occupational therapy and productivity tool, the balance board, to engage the sense of balance. Balance boards enable controlled stimulation of the vestibular sense. However, they are by no means necessary to stimulate this foundational sensory system. Pedagogical designs can stimulate the vestibular sense through everyday objects such as rocking chairs, or simply inviting rocking, tilting, spinning, or whole-body movement. Vestibular stimulation is an axis of any activity; however, most activities provide extremely limited stimulation to this system and do not leverage it as a resource for conceptual thinking. BBM offers an exploratory case of vestibular-centric, vestibular-differentiated mathematical tools.

Concluding Remarks

BBM is built to foster balance-driven mathematical conceptualizations. These constitute restructurations that reformulate traditionally graphical disciplinary representations. For example, negative numbers are restructured as counterbalances to positive numbers. Furthermore, enacting coordinated actions with peers constitutes a means to conceptualize the relations among multiple evolving quantities. Through BBM activities, children can cultivate multimodal mathematical notions, drawing upon these to animate and reason about 2-dimensional representations.

BBM activities set disciplinary representations such as graphs and number lines up as resources for controlling and communicating about dynamic balance experiences, with the balance experiences in turn informing learners' experiences of these representations. In so doing, BBM instantiates the embodied design process of drawing upon children' embodied experiences, in dialogue with disciplinary resources and practices, as the basis for math instruction (Abrahamson, 2014) and inclusive education (Abrahamson et al., 2019; Tancredi et al., 2022a, b).

BBM offers a different entry point into supporting learners' comprehension of mathematical concepts—one that makes use of learners' histories of sensorimotor experiences in the world and skill for learning new ways of moving. Additionally, experiences of balance may especially offer an additional channel of common ground for joint meaning-making across learners with different sensorial access to the world. This could have implications for creating inclusive opportunities across learners of different profiles, including nonspeaking, blind, and deaf learners. Further accessibility features, such as revised sonification and voicing and additional physical configurations such as rocking chair options, are necessary for BBM to realize its inclusive potential. Beyond BBM, explorations of balance-based conceptualizations may also open onto new somatic-driven mathematical epistemologies, towards a more sensory-expansive and sensorially inclusive mathematics.

Appendix 1. Balance Board Math activities

For interested readers, I elaborate here on additional BBM activities not described above.

Balance Number Line (BNL)

An additional dimension of the BNL is the use of magnetic arrows. Magnetic number arrows are available to document, plan, and compare movements on the number line. The purpose of these arrows is to help learners make sense of a common sticking point with equations: parsing numbers' signs and operations.

Vector arrows correspond to a specific number, such as −2 or 3. Each arrow has a length (for example, the +3 arrow in Fig. 14 has a length of 3 number line units), a color (red for negative numbers and green for positive numbers), and a default direction indicated by the head of the arrow. Each arrow is labeled with its number, including its sign, such that when the point faces its default direction, the label is upright. When adding, arrows are placed in their default orientation such that the flat end is at the existing cumulative sum (0 for a first addend) and the pointed end indicates the new cumulative sum. When subtracting, arrows are flipped horizontally before adding. The negative signifier is an embedded quality of the number arrow, while the operation of subtraction corresponds to the action of flipping an arrow,

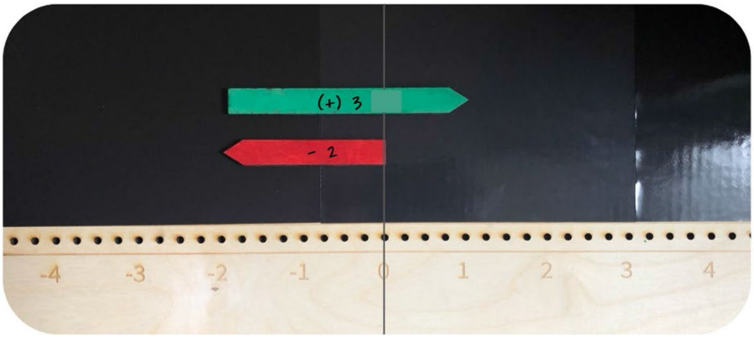


Fig. 14 Example BNL number arrows for $0+(-2)+3$

grounding the difference between the operation of subtraction and the negative sign. In practice, once introduced to placement guidelines, learners can be invited to translate expressions and equations using the magnetic arrows.

To enact an expression through balance, the learner then moves their hands along each arrow, pausing at their endpoints. Arrows and balance convey different aspects of a mathematical expression. Balance conveys only the relative distance to 0 at each step of an expression. Through challenges such as finding all arrows that can restore balance when starting at a given point on the number line, learners can reflect on what is the same across procedures such as adding 2 and subtracting negative 2.

BNL users are invited to codify their movements using the arrows and later translate the arrows to written expression and equations. For example, upon solving the moving-in-balance challenge, a child might be asked to identify some specific pairs of hand positions that balance, such as having the left hand at -1 and the right hand at 1 (position 1), or the left at -3.5 and the right at 3.5 (position 2). They are then asked to compare what arrows can get their hands from position 1 to position 2. For example, to go from -1 to -3.5, they need to subtract 2.5 (or add (-2.5)), whereas to go from 1 to 3.5, they need to add 2.5 (or subtract (-2.5)). By analyzing movement between sets of balanced pairs (positive and negative counterparts), learners can reconcile their movement solution with their prior knowledge of symbolization, establishing symmetrical arithmetic transformations. Once familiar with the board and arrow set-up, learners are invited to invent original balance-movement sequences by moving their hands along the number line. Symbolic expressions become a useful means for the child to encode these unique moves for other learners to be able to decode and reenact.

Balance Graphing (BG)—Function Exploration

In addition to the activities described above, learners also explore an introductory activity, Function Exploration. In Function Exploration (Fig. 14), users attempt to trace a focal function, for example, $\sin(x)$, by rocking on the board. The purpose of

Fig. 15 A round of Function Exploration in progress

this activity is to introduce the basic mapping of board movement to a graph and to be able to compare two functions drawn by two peers at the same time (for example, sin(x) vs. cos(x), sin(x) vs. sin(2x), or $y = x$ vs. $y = 0.5x$). As with all BG activities, the digital display is programmed to draw a graph in real-time of their board's angle. If the learners' graph follows the dotted line on the screen (within some tolerance), the line they are generating turns dark blue, and they hear a resonant sound. If they stray beyond the tolerance, their line turns green, and the sound fades to silence. When the graphs reach the end of the screen, each player's total percentage of green points is displayed on the left side. By default, the target function is displayed as a dotted line on the screen, although it can also be hidden.

For example, in Fig. 15, the child on the left is trying to trace the sine function. She leans forward as she traces a downtrending section of the function. The girl on the right tries to trace the cosine function. Her board is mostly flat as she anticipates leaning backward for an upward-trending section.

Appendix 2. Discussion of balance board types

Balance board designs vary by their application, including physical therapy and productivity. The single rocking axis of the BBM board (Fig. 16A) simplifies the vestibular stimulation provided by the activity and mirrors the kinds of spontaneous rocking exhibited by learners when rocking and fidgeting. In BBM, the experience of being in balance is important. As such, boards where resting in a balanced state are highly difficult, such as boards with a cylindrical base (Fig. 16B) are not desirable. I acknowledge, however, that a semisphere-base boards (Fig. 16C) could open additional possibilities of interest for further math concepts, not explored here.

Common usage of balance boards involves standing on a balance board and attempting to maintain the board in balance. In BBM, in contrast, the goal is for

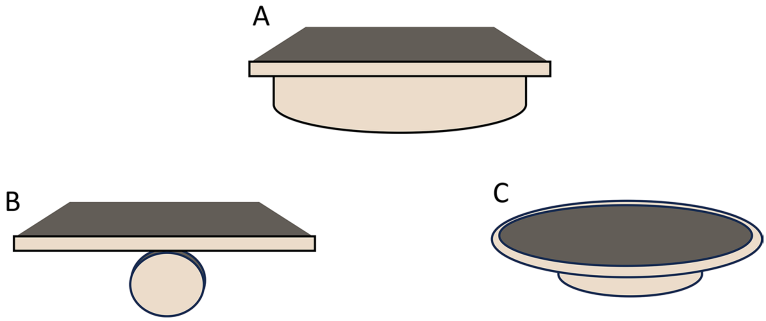


Fig. 16 Types of balance boards. Balance boards involve a flat surface with a rounded base. **A** Board with a gradually sloped base: can rock in two directions, used for BBM activities. **B** Board with a cylindrical base: rocks in only two directions. More difficult to achieve and maintain balance. **C** Board with a half-spherical base: can tilt in any direction

users to experience different angles and rocking patterns with ease. As such, rather than stand on the board, learners sit on the board's surface. In this position, rather than maintaining the head vertical, the core and head move together with the board, such that rocking the board generates consistent vestibular sensory stimulation. To support this differential usage position, I default to using a very large balance board with handle cut-outs and added protective bumpers to cushion extreme rocking. Balance Graphing activities are also compatible with other single-axis rockable objects, such as rocking chairs, and have been successfully used with assorted rocking and balancing objects.

Appendix 3. Two mapping modes of board angle to graph

Inclinometer angle data can be mapped onto graphical output in different ways. Two options of mathematical interest are angle-based and slope-based mappings (Table 3). In angle-based mapping, a given level of board tilt (*angle*) maps onto a given y position when graphing. In slope-based mapping, the *slope* of the surface of the board relative to the ground controls the local slope of the graph moment-to-moment. Whereas angle-based mapping is the default setting used in Balance Graphing activities, slope-based mapping may be of particular interest in teaching other math concepts such as derivatives. For example, in future activities, a learner could use angle-based vs. slope-based mapping to translate between a function and its first-order derivative. If a learner traces a function $f(x)$ according to a slope-based mapping (generating a graph of the angle of their board as they go), they will draw that function's first-order derivative, $f'(x)$ (Fig. 17). For example, at the first point shown in Fig. 17, the slope of $f(x)$ is 0, so the learner would make their board flat (angle 0). Thus $f'(x)$ is 0. At the second point on $f(x)$ highlighted in Fig. 17, the slope

Table 3 Comparison of angle vs. slope-based board mapping

	Angle-based mapping	Slope-based mapping
Overall	Y position corresponds to rocking angles: a given angle is a given *y* value	Each moment, the next *y* value is determined relative to the last one by the current slope of the board
Lean back (or left)	*Y* value increases	*Y* value increases (because board slope is positive)
Lean forward (or right)	*Y* value decreases	*Y* value decreases (because board slope is negative)
Board is in balance	*Y* value is 0	*Y* value remains the same as prior value
Holding board still while not in balance	*Y* value remains constant	*Y* value increases or decreases per the slope of the board's tilt

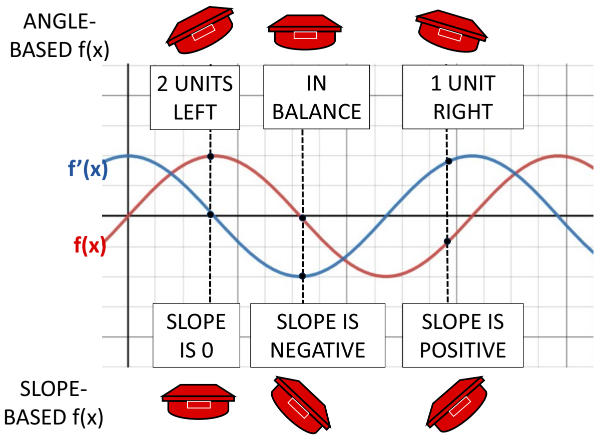


Fig. 17 Angle vs. slope-based mapping of a function; slope-based mapping supports exploring derivatives

of $f(x)$ is negative, so they lean right to match this slope. They lean two units to the right, so $f'(x)$ is -2. Thus, tasks where learners are challenged to coordinate action in angle-based and slope-based mode could develop their understanding of the relation between functions and their derivatives.

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Data Availability There are no data associated with this manuscript. A tutorial for setting up Balance Graphing and an open-access version of the activity code can be found at tinyurl.com/BBM-Balance-Graphing.

Declarations

Competing Interests The authors declare no competing interests.

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